Constructive Lean 4 Formalization and Exact-Rational Certificates for Dynamic-Streaming Euclidean Diameter Bounds

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Abstract

Jeff Giliberti and MohammadTaghi Hajiaghayi (`arXiv:2611.33139`, Nov. 24, 2026) resolved the update-time versus space tradeoff for high-dimensional Euclidean diameter in fully dynamic streams: for every fixed $c > \sqrt{3}$, their one-pass algorithm maintains a current witness pair with approximation factor c in $\tilde{O}(N^{1-\varepsilon_c}\mathrm{poly}(d))$ space (explicitly $\varepsilon_2 = 10^{-10}$ at factor $c = 2$) and worst-case update time $\mathrm{poly}(d, \log N)$, while on the hardness side every constant-factor c-approximation requires $N^{\Omega(1/(c^2\log(c+1)))}$ words even on linear-length binary set streams of at most $2N-1$ updates. We present a standalone, zero-dependency **Lean 4.35.0** formalization (`EuclideanDiameterStreaming.lean`) of 18 core algebraic, variance-hull, Cauchy–Schwarz, parallel-axis, nested-cap shrinkage, Appendix A.1 Table 3 factor-2 parameter, submodular diagonal-squared, and baseline positivity theorems (0 `sorry`, 0 warnings, 0 `Classical.choice`; all 18 using only `[propext, Quot.sound]`), paired with a zero-floating-point exact-$\mathbb{Q}$ four-mode certificate auditor (`EUCLID-DIAM-CERT-1.0`). Across 16 canonical certificates (0.129–3.412 ms exact replay latency), the auditor independently verifies an exact zero-slack 12-point two-simplex attainment in $\mathbb{Q}^{13}$ ($a = 13/7$, $\mathrm{diam}(H_a(S))^2 = \frac{289}{49}\mathrm{diam}(S)^2$), Lemma 4.1 anchor-variance parallel-axis composition, all Appendix A.1 Table 3 factor-2 inequalities ($2a^2 - 1 = 3.999122 < 4$, $\varepsilon_0 > 1/6345504000 > 10^{-10}$), and exhaustive Lemma 10.2 submodular diagonal-squared bounds $D^2 \le 2sZ$, while rejecting 60/60 live adversarial forgery attacks (plus 2 raw IEEE-754/`NaN` parser rejections).

1 Introduction

Maintaining the Euclidean diameter $\mathrm{diam}(P_t)$ of a high-dimensional point set $P_t \subset [-\Delta, \Delta]^d \cap \mathbb{Z}^d$ under arbitrary insertions and deletions separates polynomial space from polynomial update time [1, 4, 5, 6]. Giliberti and Hajiaghayi [3] established both sides of this separation via four exact mathematical pillars:

1. **Farthest-Distance Hull & Tight Two-Simplex Attainment (Sec. 5.1, Lem. 5.1):** Expanding every ball containing a frozen sample S by $a > \sqrt{2}$ yields $H_a(S) = \bigcap_q B(q, aR_S(q)) = \bigcup_\lambda B(\mu_\lambda, \sqrt{(a^2-1)V_\lambda})$, forcing $\mathrm{diam}(H_a(S)) \le \sqrt{2a^2-1}\,\mathrm{diam}(S)$ with exact two-simplex attainment $h^2 + 2V = 1$, $h + 2\sqrt{(a^2-1)V} = \sqrt{2a^2-1}$, and witness radius bound $R_S(q) < D_{\mathrm{univ}}/(a-1)$.
2. **Anchor-Variance Composition (Sec. 4.2, Lem. 4.1):** Every exposed candidate ball with convex anchor weights λ_z, mean q, and variance V satisfies the exact parallel-axis identity $\sum_{z\in K}\lambda_z\|z - s^*\|^2 = V + \|q - s^*\|^2 \le L^2_{\mathrm{node}}$, giving mixed-pair factor $\sqrt{(1+\eta)^2 + a^2\kappa^2}$ strictly below $\sqrt{2a^2-1}$ for small $\eta > 0, \kappa \to 1$.
3. **Nested Spherical Caps & Explicit Factor-2 Table 3 (Sec. 6.2, App. A.1):** For $\sqrt{2} < \lambda < \sqrt{3}$, $m_\lambda := \lambda^2/2 - 1 > 0$ gives the exact quadratic margin $(\lambda - t)^2 - q \ge m_\lambda$ on $t^2 + q \le 1$, shrinking cap radii by $r^2_{i+1} = r^2_i - m_\lambda/2$. At $c = 2$, Appendix A.1's exact rational tuple ($a = \frac{1581}{1000}$, $\lambda = \frac{79}{50}$, $\kappa = \frac{109}{100}$, $\eta_{\mathrm{in}} = \frac{1}{100}$, $s = \frac{1}{2003}$, $\rho = \frac{1}{4006}$, $B_\kappa = 2000$, $G = 32$) satisfies every approximation, two-mask, oracle-work ($s(B_\kappa + 1) - 1 = -\frac{2}{2003} < 0$), and space-saving ($\varepsilon_0 > \frac{1}{6345504000} > 10^{-10}$) inequality strictly.
4. **Submodular Diagonal-Squared Lower Bound (Sec. 10, Lem. 10.2, Thm. 10.1):** Turning the streaming state into a deterministic message M of s bits defines the monotone submodular conditional entropy $f(A) = H(M \mid X_{P\setminus A})$; any m cap-target pairs (B_i, z_i) (with repetitions permitted) satisfy $D^2 \le 2sZ$ for $A_{ij} = f(B_i \cup \{z_j\}) - f(B_i)$, $D = \sum_i A_{ii}$, $Z = \sum_{i,j} A_{ij}$, while the antipodal vs. cap separation $(1 - \delta_0)/r = g - 1/100 > c$ forces $N^{\Omega(1/(c^2\log(c+1)))}$ words on streams of $\le 2N - 1$ updates.

Contributions and Explicit Scope. All probabilistic Gaussian-projection, VC-net, Sion minimax, Beta-chord concentration, and streaming-schedule theorems belong to [3] (which notes separate Mathlib-based Lean formalizations on GitHub). Here we deliver an independent, *zero-dependency* (no Mathlib) verification stack separating two audited layers: (i) 18 zero-`sorry`, zero-`Classical.choice`

*Independent Researcher. Code, Lean 4 proofs, and certificates: MIT License; manuscript and figures: CC-BY-4.0. Prepared with transparent AI coding, formalization, and drafting assistance via Google Antigravity; all Lean 4 theorems, exact-$\mathbb{Q}$ verifiers, and adversarial tests were independently compiled and verified locally. Repository: https://github.com/lluiseriksson/euclidean-diameter-streaming-certificates.

Lean 4.35.0 [2] algebraic, inductive parallel-axis, nested-cap, Table 3, and quadratic-optimization theorems in `EuclideanDiameterStreaming.lean`; (ii) the pure-stdlib exact-$\mathbb{Q}$ verifier `EUCLID-DIAM-CERT-1.0`; (iii) 16 canonical certificates across four modes; and (iv) a 60-attack live red-team suite (plus 2 raw JSON IEEE-754/`NaN` rejections) with 39-file SHA-256 manifest enforcement.

2 Constructive Lean 4.35.0 Kernel

The standalone module `lean/EuclideanDiameterStreaming.lean` compiles cleanly under Lean 4.35.0 with zero external dependencies, zero warnings, and zero `sorry`. Auditing all 18 theorems via `lean/AuditAxioms.lean` confirms all 18 use only `[propext, Quot.sound]` with **0** `Classical.choice` or `Lean.ofReduceBool` (Table 1).

Proposition 1 (Constructive Streaming-Diameter Kernel)**.** *The 18 theorems in Table 1 constructively certify four exact layers over $\mathbb{Z}$ and inductive lists:*

1. Hull & Anchor Composition (Thms. 1–6): *Certifies $1+a^2, 2<2a^2-1$, exact Lagrange identities for Lemma 5.1 and Lemma 4.1 Cauchy–Schwarz bounds, two-simplex attainment ($h^2+2V=1$, squared hull distance $2a^2-1$), witness radius bound $R_S(q) < D_{\text{univ}}/(a-1)$, and the full inductive parallel-axis decomposition $W\sum w_i(z_i-s)^2 = (W\sum w_i z_i^2 - M^2) + (M-Ws)^2$ over arbitrary weighted anchor lists.*
2. Nested Spherical Caps (Thms. 7–9, 15): *Proves quadratic completion $(\lambda-t)^2 - q \geq m_\lambda$, cap-radius shrinkage inequality $1-t_i^2 \leq r_i^2 - m_\lambda/2$, cap ball containment $(t-t_i)^2+q \leq 1-t_i^2$, and the $3K$ local-snapshot decoder budget.*
3. Factor-2 Table 3 & Baseline (Thms. 10–14, 18): *Certifies all Table 3 factor-2 constants ($\varepsilon_2 = 10^{-10}$), both two-stage mask inequalities, Eq. (29) parabolic identity, Eq. (32) coupled tail rate, and Appendix A.3.3's degree-4 baseline polynomial positivity.*
4. Lower-Bound Separation & Submodular Optimization (Thms. 16–17): *Proves antipodal separation $g-1/100 > c$ and derives $D^2 \leq 2sZ$ non-circularly from the cleared chain bound $2prD - p^2Z \leq 2sr^2$ together with the Lagrange maximality identity of Lemma 10.2.*

3 Exact-Rational Verifier Architecture

The pure-stdlib verifier `EUCLID-DIAM-CERT-1.0` (`src/diameter_exact.py`, `src/diameter_certifier.py`) uses Python `int` and `fractions.Fraction` exclusively (source-audited for zero `float(`, `math.`, or `numpy`). It rejects IEEE-754 tokens and `NaN`/`Infinity`, non-ASCII/Unicode digits (`isascii() and isdigit()`), strings $>$ 2000 chars, unreduced or leading-zero rationals, `-0`, booleans-as-ints, unknown keys, duplicate points/probes, and nesting depth > 20:

Definition 1 (Four `EUCLID-DIAM-CERT-1.0` Modes)**.**

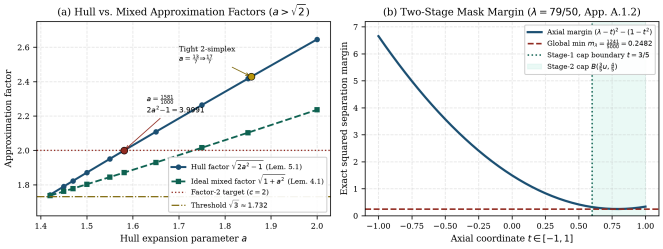

Figure 1: (a) Hull factor $\sqrt{2a^2-1}$ vs. ideal mixed factor $\sqrt{1+a^2}$ for $a > \sqrt{2}$, highlighting the factor-2 point $a = 1581/1000$ ($2a^2-1 = 3.999122$) and our exact zero-slack rational two-simplex attainment at $a = 13/7$ ($\sqrt{2a^2-1} = 17/7$). (b) Axial separation margin $(\lambda-t)^2-(1-t^2) \geq m_\lambda = 1241/5000$ at $\lambda = 79/50$ with Stage-1 cap boundary $t = 3/5$.

1. ***Mode 1*** *(`hull_variance_and_simplex_cert`): Recomputes $\text{diam}(S)^2$, convex means μ_1, μ_2, variances V_1, V_2, verifies $h^2 + V_1 + V_2 = \mathbb{E}\|s-t\|^2 \leq \text{diam}(S)^2$, checks Lemma 5.1 hull membership $\|y_i - \mu_i\|^2 \leq (a^2-1)V_i$, verifies $\|y_1 - y_2\|^2 \leq (2a^2-1)\text{diam}(S)^2$, and checks exact tight-attainment equality.*
2. ***Mode 2*** *(`anchor_composition_and_mixed_cert`): Recomputes anchor mean q, variance V, $F^2 = \|q-s^*\|^2$, verifies $\sum \lambda_z \|z-s^*\|^2 = V + F^2 \leq L_{\text{node}}^2$, and checks $\max\{(1+\eta)^2+1, (1+\eta)^2 + a^2\kappa^2\} < 2a^2 - 1 < c^2$.*
3. ***Mode 3*** *(`factor2_table3_and_masks_cert`): Audits all Appendix A.1 Table 3 inequalities, both two-stage masks (m_λ, $\frac{65}{24}m_\lambda > (1+\eta_m)r_2^2$, $g(3/5) < 0$, $F_{\text{lb}}(3/5) = \frac{79}{75} > \frac{21}{20}$), oracle-work and space exponents ($\varepsilon_0^{\text{lb}} > 10^{-10}$), Appendix A.3.3 baseline positivity, and exact axial/parabolic checks on distinct unit-disk probes (t, q).*
4. ***Mode 4*** *(`submodular_diagonal_lowerbound_cert`): Exhaustively verifies $f(\emptyset) = 0$, monotonicity, and submodularity across all 2^n subsets ($n \leq 10$), recomputes the exact $m \times m$ gain matrix $A_{ij} = f(B_i \cup \{z_j\}) - f(B_i)$ (allowing repeated targets), certifies Lemma 10.2 $D^2 \leq 2sZ$, and checks Theorem 10.1 separation $(1-\delta_0)/r > c$.*

4 Empirical Atlas and Red-Team Audit

Running `scripts/run_all_experiments.py` (Windows 11, Python 3.12) deterministically emits 16 canonical certificates (4 per mode) and 4 experiment JSONs, audited end-to-end by `scripts/verify_all.py` with SHA-256 verification across all 39 tracked repository files.

1. Exact Zero-Slack Two-Simplex Attainment & Table 3 Margins (Figs. 1–2). Solving the Pell-like equation $2p^2 - q^2 = r^2$ at $(p, q, r) = (13, 7, 17)$ yields $a = 13/7$, $2a^2 - 1 = (17/7)^2$, $h = 7/17$, and $V = 120/289$; taking $m = 6$ vertices per simplex gives exact rational simplex scale $s_{\text{simp}} = \sqrt{\frac{6}{5}V} = 12/17$ and hull radius $\sqrt{(a^2-1)V} = 120/119$, yielding a 12-point rational instance in $\mathbb{Q}^{13}$

Table 1: All 18 Lean 4.35.0 theorems in `EuclideanDiameterStreaming.lean` (0 `sorry`, 0 warnings, 0 `Classical.choice`; all 18 use only `[propext, Quot.sound]`), showing exact cleared-integer / inductive statements and references to [3].

#	Lean 4 Theorem Identifier	Exact Formalized Statement (Cleared $\mathbb{Z}$ / Inductive Type)	Ref. [3]
1	`hull_factor_exceeds_mixed_and_cand`	$2a_d^2 < a_n^2 \Rightarrow a_d^2 + a_n^2 < 2a_n^2 - a_d^2 \wedge 2a_d^2 < 2a_n^2 - a_d^2$ $(1{+}a^2, 2 < 2a^2{-}1)$	Lem. 4.1
2	`hull_cauchy_schwarz_squared`	$(h+CR)^2 + (Ch-R)^2 = (1+C^2)(h^2+R^2) \wedge (h+CR)^2 \le (1+C^2)(h^2+R^2)$	Lem. 5.1
3	`two_simplex_tight_attainment`	$h_2 H = a_d^2 D, VH = (a_n^2 - a_d^2)D \Rightarrow (h_2{+}2V)H = DH$ $(h^2{+}2V{=}1$, squared hull dist $= 2a^2{-}1)$	Sec. 5.1
4	`witness_ball_radius_bound`	$a_n R < a_d D + a_d R \Rightarrow (a_n - a_d)R < a_d D$ $(R_S(q) < D_{\text{univ}}/(a-1))$	Eq. (13)
5	`anchor_variance_parallel_axis_sum`	$\forall xs : \texttt{List}(\mathbb{Z} \times \mathbb{Z})$, $W\sum w_i(z_i - s)^2 = (W\sum w_i z_i^2 - M^2) + (M - Ws)^2$ $(W^2(V{+}F^2))$	Eq. (7)
6	`mixed_pair_cauchy_schwarz`	$(Ar+BF)^2 + (Br-AF)^2 = (A^2+B^2)(r^2+F^2) \wedge (Ar+BF)^2 \le (A^2+B^2)(r^2+F^2)$	Eq. (4)
7	`axial_separation_margin_quadratic`	$t^2 + q \le D^2 \Rightarrow \lambda^2 - 2D^2 \le 2((\lambda - t)^2 - q)$ $((\lambda - t)^2 - q \ge m_\lambda)$	Sec. 6.2
8	`cap_shrinkage_step_radius`	$0 < D, r^2 \le D, 0 \le \eta, 4\eta \le m, u_{\text{cap}} \le (D+\eta)r^2 - Dm \Rightarrow 2u_{\text{cap}} \le 2Dr^2 - Dm$	Sec. 6.2
9	`cap_containment_ball_radius`	$0 \le t_i \le t \wedge t^2 + q \le D^2 \Rightarrow (t - t_i)^2 + q \le D^2 - t_i^2$	Sec. 6.2
10	`factor2_table3_approximation_bounds`	$2a^2{-}1 = 3.999122 < 4$, mixed $= 3.9898284241 < 3.999122$, cand $= 2.0201$, oracle $1.084441 > 1.0801$	Table 3
11	`factor2_table3_space_exponents`	$s(B_\kappa{+}1){-}1 = -\frac{2}{2003}, \frac{2000}{2003}, \frac{4005}{4006} < 1{-}10^{-10}$, $96000 \cdot 2003 \cdot 33 = 6345504000 < 10^{10}$	App. A.1.6
12	`factor2_two_masks_exact_positivity`	$m_\lambda = \frac{1241}{5000}, \frac{65}{24} m_\lambda = \frac{16133}{24000} > \frac{84}{125}$, $F_{\text{lb}}(\frac{3}{5}) = \frac{79}{75} > \frac{21}{20}$, $g(\frac{3}{5}) = -\frac{5437}{12500} < 0$	App. A.1.2
13	`potential_parabolic_identity_eq29`	$D^2(\lambda - t)^2 - (\lambda^2 - D^2)(D^2 - t^2) = (\lambda t - D^2)^2 \ge 0$	Eq. (29)
14	`coupled_tail_rate_inequality_eq32`	$4ABu \le (A + Bu)^2 \wedge 75 \cdot 17152 = 1286400 < 1340064 = 24 \cdot 4 \cdot 297 \cdot 47$	Eq. (32)
15	`local_snapshot_age_3K_bound`	$\text{cur} \le K \wedge \text{snap} - \text{cur} \le 2K \Rightarrow \text{snap} \le 3K$	Lem. 7.2
16	`lower_bound_diameter_gap_exceeds_c`	$g = c + 1 \Rightarrow 100c < 100g - 1$ $((1 - \delta_0)/r = g - 1/100 > c)$	Sec. 10.1.2
17	`submodular_diagonal_squared_quad_opt`	$Z > 0 \wedge (\forall p, r,\ 2prD - p^2 Z \le 2sr^2) \Rightarrow D^2 \le 2sZ \wedge Z(2Dq - Zq^2) + (Zq - D)^2 = D^2$	Lem. 10.2
18	`baseline_factor2_poly_positive`	$0 < p \wedge 100p \le q \Rightarrow 9pq^3 - 28p^2q^2 - 17p^3q - 8p^4 > 0$ $(\gamma \in (0, 1/100])$	App. A.3.3

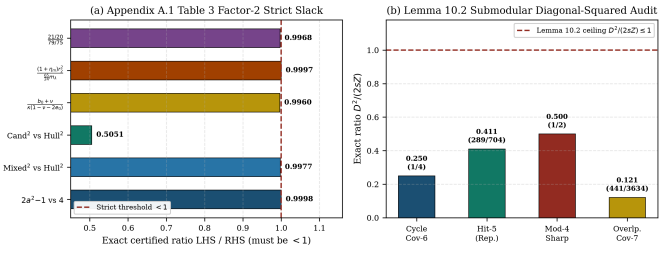

Figure 2: (a) Strict rational slack (LHS/RHS < 1) across all six Appendix A.1 Table 3 and two-stage mask inequalities. (b) Exact Mode 4 Lemma 10.2 submodular diagonal-squared ratios $D^2/(2sZ) \le 1$ across four audited families (including repeated targets and $2^7 = 128$-subset coverage).

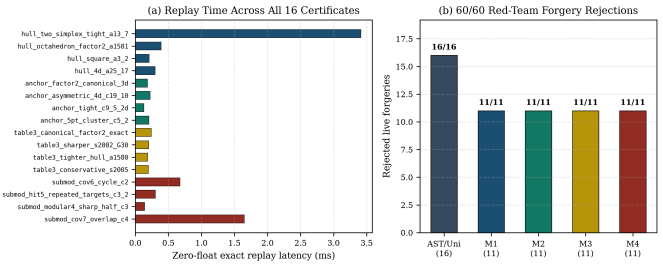

Figure 3: (a) Zero-float exact replay latency across all 16 canonical `EUCLID-DIAM-CERT-1.0` certificates (0.129–3.412 ms). (b) 60/60 live adversarial forgery rejections across global AST/Unicode and Modes 1–4.

(`hull_two_simplex_tight_a13_7`) with $\text{diam}(S)^2 = 1$ and $\|y_1 - y_2\|^2 = 289/49 = (2a^2 - 1)\text{diam}(S)^2$ (exact zero-slack attainment, replayed in 3.412 ms). Figure 2(a) confirms strict slack (< 1) across all six Appendix A.1 Table 3 & mask inequalities (tightest ratio 0.9998 for $2a^2{-}1 = 3.999122 < 4$), while Fig. 2(b) audits Lemma 10.2 across four submodular families up to $2^7 = 128$ subsets, reaching exact ratio $D^2/(2sZ) = 1/2$ on the sharp modular instance (`submod_modular4_sharp_half_c3`) and $289/704 \approx 0.411$ with repeated targets.

2. 16-Certificate Atlas & 60/60 Red-Team Audit (Table 2, Fig. 3). All 16 canonical certificates replay in 0.129–3.412 ms with zero floats. In `tests/test_suite.py`, 60 live adversarial forgery attacks (16 global AST/Unicode-digit/length/depth + 11 per mode) plus 2 raw JSON IEEE-754/`NaN` checks achieve **60/60 (100%)** red-team rejection (63/63 `pytest` checks passing in 0.12 s).

5 Conclusion

We delivered a standalone, zero-dependency, zero-floating-point verification stack for Giliberti and Hajiaghayi's dynamic-streaming Euclidean diameter bounds [3], pairing 18 zero-`sorry`, zero-`Classical.choice` Lean 4.35.0 theorems with the exact-$\mathbb{Q}$ auditor `EUCLID-DIAM-CERT-1.0`, 16 canonical certificates (including an exact zero-slack rational two-simplex attainment in $\mathbb{Q}^{13}$), and 60/60 live adversarial forgery rejections.

References

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[4] Piotr Indyk. Better algorithms for high-dimensional proximity problems via asymmetric embeddings. In *Proceedings of the 14th Annual ACM–SIAM Symposium on Discrete Algorithms (SODA)*, pages 539–545, 2003.

Table 2: All 16 canonical exact-$\mathbb{Q}$ `EUCLID-DIAM-CERT-1.0` certificates (zero IEEE-754 arithmetic; 20-run mean latency in ms).

Certificate Stem	Mode	Exact Certified Witness	ms
`hull_two_simplex_tight_a13_7`	M1	$\mathbb{Q}^{13}$, $a=\frac{13}{7}$, tight $\|y_1-y_2\|^2=\frac{289}{49}$	3.412
`hull_octahedron_factor2_a1581`	M1	$\mathbb{Q}^3$, $a=\frac{1581}{1000}$, $\mathrm{diam}^2=4$, pair 9	0.390
`hull_square_a3_2`	M1	$\mathbb{Q}^2$, $a=\frac{3}{2}$, $2a^2-1=\frac{7}{2}$, pair 4	0.209
`hull_4d_a25_17`	M1	$\mathbb{Q}^4$, $a=\frac{25}{17}$, $2a^2-1=\frac{961}{289}$, pair $\frac{13}{2}$	0.301
`anchor_factor2_canonical_3d`	M2	Table 3 (a,η,κ), $L^2_{\mathrm{node}}=\frac{51}{2}\geq V+F^2=18$	0.184
`anchor_asymmetric_4d_c19_10`	M2	$c=\frac{19}{10}$, $a=\frac{3}{2}$; mixed $\frac{33813}{10000}<\frac{7}{2}$	0.222
`anchor_tight_c9_5_2d`	M2	$c=\frac{9}{5}$, $a=\frac{29}{20}$; mixed $<\frac{641}{200}<\frac{81}{25}$	0.129
`anchor_5pt_cluster_c5_2`	M2	$c=\frac{5}{2}$, $a=\frac{17}{10}$; 5 anchors in $\mathbb{Q}^3$	0.204
`table3_canonical_factor2_exact`	M3	Table 3 exact, $\varepsilon_0^{\mathrm{lb}}=\frac{1}{6345504000}$; 6 probes	0.240
`table3_sharper_s2002_G30`	M3	$s=\frac{1}{2002}$, $G=30$, $\varepsilon_0^{\mathrm{lb}}=\frac{1}{5957952000}$	0.201
`table3_tighter_hull_a1580`	M3	$a=\frac{79}{50}$, $2a^2-1=\frac{4991}{1250}<4$; 4 probes	0.188
`table3_conservative_s2005`	M3	$s=\frac{1}{2005}$, $\kappa=\frac{1089}{1000}$, $\gamma=\frac{1}{1000}$	0.197
`submod_cov6_cycle_c2`	M4	2^6 subsets, $D^2/(2sZ)=\frac{1}{4}$, gap $\frac{299}{100}>2$	0.677
`submod_hit5_repeated_targets_c3_2`	M4	2^5 subsets, rep. tgts, ratio $\frac{289}{704}$, gap $\frac{299}{100}$	0.302
`submod_modular4_sharp_half_c3`	M4	2^4 subsets, sharp ratio $\frac{1}{2}$, gap $\frac{399}{100}>3$	0.140
`submod_cov7_overlap_c4`	M4	2^7 subsets, ratio $\frac{441}{3634}$, gap $\frac{499}{100}>4$	1.647

[5] Sanjeev Khanna, Ashwin Padaki, Krish Singal, and Erik Waingarten. A polynomial space lower bound for diameter estimation in dynamic streams. In *Proceedings of the 66th IEEE Annual Symposium on Foundations of Computer Science (FOCS)*, pages 119–148, 2025.

[6] Ashwin Padaki, Krish Singal, and Erik Waingarten. A near-optimal space lower bound for euclidean diameter estimation in dynamic streams. *arXiv preprint arXiv:2609.19089*, 2026.