A channel-by-channel check of the $K\bar{K}\eta$ prediction for $X(2370)$

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Abstract

We reconstruct the seven charge-summed ratios in Wang's calculation for a pseudoscalar parent. The listed widths give $\Gamma(K\bar{K}\eta)/\Gamma(K\bar{K}\pi) = 0.0134672$, while the summary gives 0.135. All six other entries agree at the printed precision, including the normalization. We derive the extrema over the quoted mixing-parameter ranges and exhibit a spectator dependence in the printed phase-space formula. These checks isolate the channel and fix the benchmark for a future measurement within this model.

Which channel changes?

We use the isospin-averaged inputs and constant amplitudes of Wang [1], Eqs. (19)–(23). Summing the distinct charge states in Eqs. (27)–(28) gives $2(1.8051\,10^{-10})/[2(8.9358 + 4.4679)10^{-9}] = 0.01346718$. As a second calculation we integrate the squared amplitudes, including the identical-particle factorials, at parent mass $M = 2.359$ GeV. The denominator in every column is the sum of the four $K\bar{K}\pi$ widths.

Final state	Printed Eq. (30)	From listed widths	Invariant integral
$\pi\pi\eta$	.641	.640845	.640866
$\pi\pi\eta'$	.226	.225923	.225927
$K\bar{K}\eta$	.135	.013467	.013467
$K\bar{K}\eta'$	.193	.192708	.192703
$K\bar{K}\pi$	1	1	1
$\eta\eta\eta$	.010	.010231	.010231
$\eta\eta\eta'$	.030	.030310	.030310

The experimental comparison in BESIII [4], Table 1, concerns $K\bar{K}\eta'$ and $K\bar{K}\pi$. The η and η' observables must retain separate labels. The present correction changes an inclusive model prediction. Deducing a parent composition or a measured branching fraction would require additional evidence.

Why the mixing uncertainties matter

Write $q = f_q/f_\pi$, $f = f_s/f_\pi$, $k = (2m_K^2 - m_\pi^2)/(27.30\, m_\pi^2)$, and let I_c be the invariant phase-space integral for $K\bar{K}c$. At fixed masses the normalized ratio is

$$R_\eta = \frac{2I_\eta}{3I_\pi} A^2, \qquad A = q\cos\phi/\sqrt{2} - kf\sin\phi.$$

The orthogonal amplitude $B = q\sin\phi/\sqrt{2} + kf\cos\phi$ replaces A for η'. In particular $A^2 + B^2 = q^2/2 + (kf)^2$. The partial cancellation in A makes its sensitivity to the mixing inputs a relevant check.

Feldmann, Kroll and Stech [2], Eqs. (3.11) and (3.15), quote $\phi = 39.3^\circ \pm 1.0^\circ$, $q = 1.07 \pm 0.02$, and $f = 1.34 \pm 0.06$. On the Cartesian product of those ranges, A increases with q and decreases with both f and ϕ. Its extrema therefore occur at opposite corners: -0.265396 and -0.123747. Squaring in reverse order gives the envelope $0.00547873 \le R_\eta \le 0.0252000$ (rounded). It assigns no joint confidence level. Simultaneously scaling all three quoted half-widths first permits a zero at scale 2.804 and first reaches 0.1345, the lower rounding boundary for 0.135, at scale 5.684. These factors describe the chosen parameter box.

A spectator check on the phase-space measure

Let $s = (p_a + p_b)^2$, u be the pair-rest-frame momentum of a, and v the parent-rest-frame momentum of c. Angular integration of invariant phase space gives $I = \int uv\, ds/\sqrt{s}$ up to a common factor [3]. Eq. (24) of [1], with the momenta defined there, instead yields $J_{ab} = \int uv\, ds/s$. Thus J_{ab}/I is the invariant-weighted mean of $1/\sqrt{s}$, between $1/(M - m_c)$ and $1/(m_a + m_b)$. As M decreases to threshold these bounds coincide. Consequently J_{ab}/J_{ac} tends to $(m_a+m_c)/(m_a+m_b)$ for positive masses, showing a dependence on the pairing when $m_b \neq m_c$. At the stated M, for $K\bar{K}\pi$ both invariant pairings give 0.730421 in the script convention. The literal formula gives 0.476601 or 0.608249. This comparison establishes which measure reproduces the listed ratios. Author code was unavailable for this reconstruction.

Model assessments and review context. Codex assisted in analysis and drafting. The scripts compare integration variables and particle pairings. Neither constitutes independent peer review. Any assessment of the final PDF belongs to its exact version record.

Primary sources. [1] Z.-G. Wang, Eur. Phys. J. C (2026). [2] T. Feldmann, P. Kroll and B. Stech, Phys. Rev. D 58, 114006 (1998). [3] Particle Data Group, Kinematics (2023), Eq. (49.20). [4] BESIII Collaboration, arXiv:2607.20366, Table 1.