Sharp marginal-erasure bounds and positive completion of nonunital quantum channels

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Abstract

We determine the exact Frobenius conditioning of deleting the traceless part of one local matrix algebra on $\mathbb{C}^m \otimes \mathbb{C}^k$. For $1 \le r < m$, the inverse constants are $\sqrt{m/(m-r)}$ for two states of rank at most r, and $\sqrt{(m+r)/(m-r)}$ for one such state against an arbitrary density matrix. Both are attained, independently of $k \ge 2$; at rank m exact collisions occur. The one-sided estimate follows from a scalar optimization combining positivity, trace control and the partial-trace norm. It yields a noise and spectral-tail bound for convex positive completion. For normalized Choi matrices, the retained coordinates are the action of a trace-preserving channel on traceless inputs. Consequently this action identifies every channel of Kraus rank below its input dimension among all channels, without a unitality assumption. For square channels we express the same conclusion through a complete exact unitary 2-design of average gate fidelities. The rank threshold is sharp, whereas optimal conditioning within the trace-preserving subclass is left open. The fidelity frame identities and general kernel/inertia method are prior work; no compressed-sampling or new randomized-benchmarking noise guarantee is asserted.

1 Question, contribution and antecedents

Average gate fidelities with unitary reference gates do not linearly determine an arbitrary quantum channel: they miss its translation, or action on the maximally mixed input. Positivity can nevertheless make that missing part unique when the channel has sufficiently small Kraus rank. We quantify this mechanism by first solving an unrestricted state-space problem exactly.

Let $\mathcal{S}_{m,k}$ denote the density matrices on $\mathbb{C}^m \otimes \mathbb{C}^k$, and set

$$\mathcal{K} = \{I_m \otimes B : B \in \mathrm{Herm}(\mathbb{C}^k),\ \mathrm{Tr}\, B = 0\}.$$

Here $\mathcal{K}$ is the traceless Hermitian part of a local algebra, not itself an algebra under multiplication. Let P be its Frobenius-orthogonal projection and $Q = \mathrm{id} - P$. Our principal result computes the largest ratio $\|\rho - \sigma\|_{\mathrm{F}} / \|Q(\rho - \sigma)\|_{\mathrm{F}}$ under either one or two rank constraints. This distinction matters: positivity-constrained reconstruction usually produces a competitor with unrestricted rank.

The reduction of low-rank identification to the inertia of a measurement kernel is classical in state tomography [1] and channel tomography [2]. In particular, Gutoski and Johnston construct small families of interactive observables identifying low-Kraus-rank channels among all channels. We do not improve their observable counts; we study the conditioning of a particular fixed omitted subspace. Kimmel et al. recover the unital block using randomized benchmarking [3]; their single-qubit complete-positivity lemma has a corrected proof [4]. Roth et al. provide a unitary-design frame for unital trace-preserving maps and compressed recovery guarantees for quantum gates [5]. We use the classical second-moment identity explicitly and separately from our completion inequality.

The proposed contribution is the pair of exact local-erasure constants, their state attainers, and the resulting quantitative completion statement without a unitality promise. The elementary proof is included in full. These comparisons delimit the claim; they do not certify worldwide priority.

2 Exact conditioning in state space

All matrix spaces below are real Hilbert spaces of Hermitian matrices with inner product $\mathrm{Tr}(XY)$. The Frobenius norm is $\|X\|_{\mathrm{F}}$. A zero denominator and nonzero numerator mean an infinite ratio. Write $\mathcal{S}_{m,k}^{(r)}$ for states of rank at most r.

Theorem 1 (Exact local-erasure constants)**.** *Let $m \geq 1$, $k \geq 2$ and $r \geq 1$ be integers. For $1 \leq r < m$,*

$$\sup_{\substack{\rho,\sigma\in\mathcal{S}_{m,k}^{(r)}\\ \rho\neq\sigma}} \frac{\|\rho-\sigma\|_{\mathrm{F}}}{\|Q(\rho-\sigma)\|_{\mathrm{F}}} = \sqrt{\frac{m}{m-r}}, \tag{1}$$

$$\sup_{\substack{\rho\in\mathcal{S}_{m,k}^{(r)},\ \sigma\in\mathcal{S}_{m,k}\\ \rho\neq\sigma}} \frac{\|\rho-\sigma\|_{\mathrm{F}}}{\|Q(\rho-\sigma)\|_{\mathrm{F}}} = C_{m,r} := \sqrt{\frac{m+r}{m-r}}. \tag{2}$$

Both suprema are attained. For $r \geq m$ both are infinite. Thus $m = 1$ has no finite positive-rank regime.

We first record the projection and the standard inertia reduction. The notation $n_+(D)$ and $n_-(D)$ counts positive and negative eigenvalues with multiplicity.

Lemma 2. *For any Hermitian D,*

$$PD = \frac{I_m}{m} \otimes \left(\mathrm{Tr}_A D - \frac{\mathrm{Tr}\, D}{k} I_k\right).$$

In particular, for $\mathrm{Tr}\, D = 0$,

$$PD = \frac{I_m}{m} \otimes \mathrm{Tr}_A D, \qquad \|PD\|_{\mathrm{F}}^2 = \frac{1}{m}\,\|\mathrm{Tr}_A D\|_{\mathrm{F}}^2. \tag{3}$$

For arbitrary matrices X, $\|\mathrm{Tr}_A X\|_{\mathrm{F}} \leq \sqrt{m}\,\|X\|_{\mathrm{F}}$. If $D = \rho - \sigma$ with $\mathrm{rank}\,\rho \leq r$, then $n_+(D) \leq r$. If also $\mathrm{rank}\,\sigma \leq r$, then $n_-(D) \leq r$.

Proof. The proposed PD belongs to $\mathcal{K}$, and its difference from D is orthogonal to every $I_m \otimes B$ with $\mathrm{Tr}\, B = 0$, by the defining identity of partial trace. This proves the projection formula. For the norm estimate, in a product basis Cauchy–Schwarz gives

$$\sum_{i,j}\left|\sum_a X_{ai,aj}\right|^2 \leq m \sum_{a,i,j} |X_{ai,aj}|^2 \leq m\,\|X\|_{\mathrm{F}}^2.$$

If a positive eigenspace of $\rho - \sigma$ had dimension greater than r, it would meet $\ker\rho$ nontrivially. On that kernel its quadratic form is nonpositive, a contradiction. Changing sign proves the last claim. □

Proof of Theorem 1. Let $D \neq 0$ be Hermitian with $\mathrm{Tr}\, D = 0$, and write its spectral decomposition as $D = D_+ - D_-$ with orthogonal positive parts. Set

$$\tau = \mathrm{Tr}\, D_+ = \mathrm{Tr}\, D_- > 0, \qquad a^2 = \|D_+\|_{\mathrm{F}}^2, \quad b^2 = \|D_-\|_{\mathrm{F}}^2.$$

If $n_+(D) \le r$, then $a^2 \ge \tau^2/r$. The positive matrices $R = \mathrm{Tr}_A\, D_+$ and $S = \mathrm{Tr}_A\, D_-$ both have trace τ. Positivity implies $\mathrm{Tr}(RS) \ge 0$ and $\|R\|_{\mathrm{F}}^2, \|S\|_{\mathrm{F}}^2 \le \tau^2$. Lemma 2 also gives $\|S\|_{\mathrm{F}}^2 \le mb^2$. Consequently

$$\|\mathrm{Tr}_A\, D\|_{\mathrm{F}}^2 = \|R - S\|_{\mathrm{F}}^2 \le \tau^2 + \min\{\tau^2, mb^2\}. \tag{4}$$

No commutation or orthogonality of the two marginals is assumed.

Put $t = b^2/\tau^2$. Equations (3)–(4) yield

$$\frac{\|PD\|_{\mathrm{F}}^2}{\|D\|_{\mathrm{F}}^2} \le \frac{1/m + \min\{1/m, t\}}{1/r + t} \le \frac{2r}{m+r}. \tag{5}$$

Indeed the middle expression is increasing for $0 \le t \le 1/m$ and decreasing for $t \ge 1/m$, since $r < m$. Its maximum is at $t = 1/m$. Orthogonality of P and Q now proves (2) as an upper bound for every difference in its stated domain.

For sharpness, choose a rank-r input projector R_0 and orthogonal output unit vectors u, v. The states

$$\rho = \frac{R_0}{r} \otimes |u\rangle\langle u|, \qquad \sigma = \frac{I_m}{m} \otimes |v\rangle\langle v| \tag{6}$$

have ranks r, m. For their difference,

$$\|D\|_{\mathrm{F}}^2 = \frac{1}{r} + \frac{1}{m}, \quad \|PD\|_{\mathrm{F}}^2 = \frac{2}{m}, \quad \|QD\|_{\mathrm{F}}^2 = \frac{1}{r} - \frac{1}{m}.$$

They attain (2) and all bounds in (5).

If both inertia indices are at most r, then $a^2, b^2 \ge \tau^2/r$. Instead of (4) use

$$\|\mathrm{Tr}_A\, D\|_{\mathrm{F}}^2 \le 2\tau^2 \le r(a^2 + b^2).$$

The hidden fraction is at most r/m, which proves the upper bound in (1). It is attained by

$$\rho = \frac{R_0}{r} \otimes |u\rangle\langle u|, \qquad \sigma = \frac{R_0}{r} \otimes |v\rangle\langle v|,$$

for which $\|D\|_{\mathrm{F}}^2 = 2/r$ and $\|PD\|_{\mathrm{F}}^2 = 2/m$. Finally, take $R_0 = I_m$ and $r = m$. These distinct rank-m states have $QD = 0$. They remain admissible for every $r \ge m$, establishing the infinite regime, including $m = 1$. □

Every nonzero member of $\mathcal{K}$ has at least m positive and m negative eigenvalues. Thus the qualitative threshold also follows immediately from the classical kernel/inertia criterion. The estimates above quantify the angle between that kernel and the relevant difference cones, rather than merely showing they do not intersect.

3 Approximate rank and convex completion

For a density matrix ρ with eigenvalues in decreasing order, define $\tau_r(\rho) = \sum_{i>r} \lambda_i(\rho)$.

Proposition 3 (Spectral tails)**.** *For $1 \le r < m$, every pair of density matrices ρ, σ obeys*

$$\|\sigma - \rho\|_{\mathrm{F}} \le C_{m,r}\, \|Q(\sigma - \rho)\|_{\mathrm{F}} + \sqrt{2}(C_{m,r} + 1)\tau_r(\rho). \tag{7}$$

Let $\mathcal{F}$ be any compact convex subset of density matrices containing ρ. If $y = Q\rho + e$, $\|e\|_{\mathrm{F}} \le \epsilon$, then every minimizer $\widehat{\rho}$ of $\|Q\sigma - y\|_{\mathrm{F}}$ over $\sigma \in \mathcal{F}$ satisfies

$$\|\widehat{\rho} - \rho\|_{\mathrm{F}} \le 2C_{m,r}\epsilon + \sqrt{2}(C_{m,r} + 1)\tau_r(\rho). \tag{8}$$

Proof. Let $s = 1 - \tau_r(\rho) > 0$, and let ρ_r be the top-r spectral truncation of ρ divided by s. At an eigenvalue tie any spectral choice is allowed. Then

$$\|\rho - \rho_r\|_{\mathrm{F}}^2 = \tau_r(\rho)^2 \sum_{i \le r} \left(\frac{\lambda_i}{s}\right)^2 + \sum_{i>r} \lambda_i^2 \le 2\tau_r(\rho)^2.$$

Apply (2) to ρ_r, σ, then use the triangle inequality and the contraction property of Q to obtain (7). A minimizer exists by compactness. Its residual is at most that of the feasible truth, hence $\|Q(\widehat{\rho} - \rho)\|_{\mathrm{F}} \le 2\epsilon$. Substitution proves (8). □

The truncation ρ_r need not belong to $\mathcal{F}$. The proof is valid because (2) compares it with every density matrix. This detail matters for Choi matrices: normalized spectral truncation does not in general preserve trace preservation. For example, the $3 \to 2$ channel with Kraus operators

$$K_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \qquad K_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

has Choi eigenvalues $2/3, 1/3$. Its normalized top-one truncation is the state of $(|00\rangle + |11\rangle)/\sqrt{2}$, whose input marginal is $\mathrm{diag}(1/2, 1/2, 0)$ rather than $I_3/3$. Convexity makes the optimization a convex problem; the proof of the error bound uses compactness and feasibility, without imposing a rank constraint on the minimizer.

4 Channel completion from the traceless-input action

Let $\Phi : M_m \to M_k$ be completely positive and trace preserving (CPTP). Use input–output tensor order and the normalized Choi matrix

$$J_\Phi = (\mathrm{id} \otimes \Phi)(|\Omega_m\rangle\langle\Omega_m|), \qquad |\Omega_m\rangle = \frac{1}{\sqrt{m}} \sum_{a=0}^{m-1} |a, a\rangle.$$

Then $J_\Phi \ge 0$, $\mathrm{Tr}\, J_\Phi = 1$, $\mathrm{Tr}_{\mathrm{out}}\, J_\Phi = I_m/m$, and $\mathrm{rank}\, J_\Phi$ is the minimum Kraus rank.

Choose Frobenius-orthonormal bases F_i and G_j of the real spaces of traceless Hermitian input and output matrices. Define

$$T(\Phi)_{ji} = \mathrm{Tr}[G_j \Phi(F_i)].$$

These bases can contain imaginary entries. The input transpose in

$$\mathrm{Tr}[J_\Phi (F_i^{\mathsf{T}} \otimes G_j)] = \frac{1}{m} T(\Phi)_{ji} \tag{9}$$

is essential. It follows by expanding $J_\Phi = m^{-1} \sum_{a,b} |a\rangle\langle b| \otimes \Phi(|a\rangle\langle b|)$.

For two TP maps the identity and input-only coefficients of $D = J_\Phi - J_\Psi$ vanish. The projection P removes precisely the output-only traceless coefficients, so orthonormal expansion gives

$$\|Q(J_\Phi - J_\Psi)\|_{\mathrm{F}} = \frac{1}{m} \|T(\Phi) - T(\Psi)\|_{\mathrm{F}}. \tag{10}$$

Thus T is exactly the retained nonconstant Choi information. The omitted translation is $\Phi(I_m/m)$.

Theorem 4 (Low-Kraus-rank completion)**.** *If* $\mathrm{rank}\, J_\Phi \le r < m$*, then for every CPTP* $\Psi : M_m \to M_k$*,*

$$\|J_\Phi - J_\Psi\|_{\mathrm{F}} \le \frac{C_{m,r}}{m} \|T(\Phi) - T(\Psi)\|_{\mathrm{F}}. \tag{11}$$

In particular $T(\Phi)$ identifies Φ among all CPTP maps. The uniform rank threshold $r < m$ is sharp for every $k \geq 2$. For $T_{\mathrm{obs}} = T(\Phi) + N$ with $\|N\|_{\mathrm{F}} \leq \delta$, every least-squares fit over all CPTP maps satisfies, for every $1 \leq r < m$,

$$\left\| J_{\widehat{\Phi}} - J_{\Phi} \right\|_{\mathrm{F}} \leq \frac{2C_{m,r}}{m}\delta + \sqrt{2}(C_{m,r}+1)\tau_r(J_{\Phi}). \tag{12}$$

Proof. Combine Theorem 1 and (10). For the endpoint consider the pure-output replacers $\Phi_u(X) = \mathrm{Tr}(X)|u\rangle\langle u|$. Each has $T = 0$ and Choi rank m; distinct output states give distinct channels with identical data.

For the noisy result, the feasible Choi matrices form a compact convex set with marginal I_m/m. Insert T_{obs}/m into the correlation coordinates, together with the known identity coefficient and zero input-only coefficients. This is $QJ_{\Phi} + e$ with $\|e\|_{\mathrm{F}} \leq \delta/m$. The residual problem differs from the displayed least-squares problem only by the factor $1/m$, so Proposition 3 applies. □

The state-space constants are exact; (11) is a valid upper bound on the smaller fixed-marginal class, not an optimal channel constant proved here. In (6) the first input marginal is R_0/r, so that attainer is not TP for $r < m$. For rectangular maps some rank classes are empty: r Kraus operators of size k by m require $rk \geq m$. For square maps Kraus rank one means a unitary channel, whose translation is zero.

4.1 Why one must not impose unitality

For square dimensions let the conventional unital part be the Hermiticity-preserving TP map

$$\Phi^{\mathrm{u}}(X) = \Phi(X) - \frac{\mathrm{Tr}\, X}{m}(\Phi(I_m) - I_m). \tag{13}$$

It fixes I_m and agrees with Φ on traceless inputs.

Corollary 5. *If $\Phi : M_m \to M_m$ is nonunital and has Kraus rank less than m, then Φ^{u} is not completely positive.*

Proof. Otherwise Φ^{u} would be a distinct CPTP channel with $T(\Phi^{\mathrm{u}}) = T(\Phi)$, contradicting Theorem 4. □

For a concrete family, let $m \geq 3$ and

$$K_0 = \mathrm{diag}(1, \ldots, 1, 3/5), \qquad K_1 = (4/5)|0\rangle\langle m-1|.$$

The two linearly independent Kraus operators satisfy $\sum_a K_a^\dagger K_a = I_m$ and define a nonunital channel of rank two. Writing $B = \Phi(I_m) - I_m = (16/25)(|0\rangle\langle 0| - |m-1\rangle\langle m-1|)$, its unital-part Choi matrix is $J_{\Phi^{\mathrm{u}}} = J_{\Phi} - I_m \otimes B/m^2$. For any $1 \leq i \leq m-2$,

$$\langle i, 0|J_{\Phi^{\mathrm{u}}}|i, 0\rangle = -\frac{16}{25m^2} < 0.$$

This illustrates the failure of an unjustified unital constraint. It is not a claim that higher-dimensional failure of complete positivity was previously unknown. There is no conflict with the corrected qubit lemma of [4]: below the qubit threshold the only square channels are unitary, and the interior index in this example does not exist for $m = 2$.

5 Complete average-gate-fidelity data

In this section input and output dimensions both equal $m \geq 2$. Let $U_1, \ldots, U_N$ be a finite exact unitary 2-design with uniform weights, and put

$$f_\ell(\Phi) = F_{\text{avg}}(U_\ell, \Phi) = \int \langle\psi|U_\ell^\dagger \Phi(|\psi\rangle\langle\psi|)U_\ell|\psi\rangle \, d\psi, \qquad \|z\|_{\text{rms}}^2 = \frac{1}{N}\sum_\ell |z_\ell|^2.$$

For a weighted design the same statements use positive weights w_ℓ with $\sum_\ell w_\ell = 1$ and the matched norm $\|z\|_{\text{rms},w}^2 = \sum_\ell w_\ell |z_\ell|^2$. No random subset of this design is used.

Lemma 6 (Classical design frame, with normalization)**.** *For any two CPTP maps on M_m,*

$$\|Q(J_\Phi - J_\Psi)\|_{\text{F}} = (m+1)\sqrt{m^2-1}\,\|f(\Phi) - f(\Psi)\|_{\text{rms}}. \tag{14}$$

Proof. Set $p = m^2 - 1$ and use the same orthonormal traceless basis F_i on input and output. Let $O(U)_{ji} = \text{Tr}[F_j U F_i U^\dagger]$. The standard pure-state second moment $\int |\psi\rangle\langle\psi|^{\otimes 2} d\psi = (I + \text{SWAP})/[m(m+1)]$ gives, for a TP map,

$$F_{\text{avg}}(U, \Phi) = \frac{1}{m} + \frac{\text{Tr}[O(U)^{\mathsf{T}} T(\Phi)]}{m(m+1)}. \tag{15}$$

For completeness the design second moment follows from

$$\int (UF_iU^\dagger) \otimes (UF_bU^\dagger)\, dU = \frac{\delta_{ib}}{p}\left(\text{SWAP} - \frac{I}{m}\right).$$

The Haar twirl lies in the span of I and SWAP; its trace is zero and its trace against SWAP is δ_{ib}, which fixes these coefficients. Pairing against $F_j \otimes F_a$ yields

$$\frac{1}{N}\sum_\ell O(U_\ell)_{ji} O(U_\ell)_{ab} = \frac{\delta_{ja}\delta_{ib}}{p}.$$

A unitary 2-design reproduces this polynomial moment. Apply this identity to $\Delta T = T(\Phi) - T(\Psi)$ in (15):

$$\|f(\Phi) - f(\Psi)\|_{\text{rms}}^2 = \frac{\|\Delta T\|_{\text{F}}^2}{m^2(m+1)^2(m^2-1)}.$$

Equation (10) proves the claim. □

The frame mechanism is the established unitary-design tomography of [5], with conventions displayed to prevent a norm or dimension factor from being imported incorrectly.

Corollary 7 (Positive completion from complete design fidelities)**.** *If Φ has Kraus rank at most $r < m$, then its complete exact-design fidelities identify it among all CPTP maps. For every such competitor,*

$$\|J_\Phi - J_\Psi\|_{\text{F}} \leq C_{m,r}(m+1)\sqrt{m^2-1}\|f(\Phi) - f(\Psi)\|_{\text{rms}}.$$

If $f_{\text{obs}} = f(\Phi) + e$ and $\|e\|_{\text{rms}} \leq \epsilon$, every minimizer of $\|f(\Psi) - f_{\text{obs}}\|_{\text{rms}}$ over all CPTP maps obeys

$$\begin{aligned}\left\|J_{\widehat{\Phi}} - J_\Phi\right\|_{\text{F}} &\leq 2C_{m,r}(m+1)\sqrt{m^2-1}\,\epsilon \\ &\quad + \sqrt{2}(C_{m,r}+1)\tau_r(J_\Phi).\end{aligned} \tag{16}$$

The uniform rank threshold is sharp.

Proof. The exact-rank statement follows from Theorem 4 and Lemma 6. A residual minimizer has $\|f(\widehat{\Phi}) - f(\Phi)\|_{\mathrm{rms}} \leq 2\epsilon$; apply (7) and (14) for the approximate-rank statement. Replacers have rank m and $f_\ell = 1/m$ for every reference unitary, establishing the endpoint obstruction. □

There is no linear reconstruction of the translation in these fidelities. The additional conclusion is a nonlinear uniqueness and stability consequence of positivity and rank. The feasible set in Corollary 7 is all CPTP maps; imposing a unital constraint would discard the nonunital examples above.

6 Verification, scope and open questions

The analytic proofs establish statements in all declared dimensions. Separate finite exact-arithmetic controls check the projection, attainers, coherent directions, complex Hermitian basis transposes, rectangular channels and spectral tails. Such controls are diagnostics, not substitutes for the proofs. The derivation and this combined manuscript have different review scopes; no score from an earlier manuscript is assigned to this one.

Three limitations are central. First, the exact constants are for unrestricted density matrices; optimal constants on the fixed-input- marginal Choi class remain open. Second, complete-design fidelities may be numerous. We do not show that a small random Clifford subset has the required uniform one-sided conditioning for arbitrary low-Kraus-rank nonunital channels. Third, a deterministic perturbation bound for calibrated fidelities is not a SPAM-robust experimental protocol. Noise extraction, gauge and calibration must be justified separately; the guarantees of [3, 5] do not automatically transfer to a larger feasible class.

These distinctions suggest concrete further problems: determine the optimal fixed-marginal constants, find sparse fidelity frames with uniform positive-completion guarantees, and obtain sharper spectral-tail terms under additional channel structure. The present paper establishes the local-erasure law and its complete-data consequence only.

AI and responsibility disclosure. The named author directed OpenAI Codex assistance in derivation, writing and exact arithmetic. Requested GPT-6 Astra xhigh audits examined component arguments with reused context and shared files. Runtime identity and isolation were not independently attested. No independent human review, Lean formalization, certified worldwide originality or predicted assessment score is claimed. A full assessment of this exact combined manuscript has not yet been obtained.

References

[1] C. Carmeli, T. Heinosaari, J. Schultz and A. Toigo, Tasks and premises in quantum state determination, *J. Phys. A: Math. Theor.* **47**, 075302 (2014). arXiv:1308.5502.

[2] G. Gutoski and N. Johnston, Process tomography for unitary quantum channels, *J. Math. Phys.* **55**, 032201 (2014). doi:10.1063/1.4867625.

[3] S. Kimmel, M. P. da Silva, C. A. Ryan, B. R. Johnson and T. Ohki, Robust extraction of tomographic information via randomized benchmarking, *Phys. Rev. X* **4**, 011050 (2014). doi:10.1103/PhysRevX.4.011050.

[4] S. Kimmel et al., Erratum: Robust extraction of tomographic information via randomized benchmarking, *Phys. Rev. X* **6**, 029902 (2016). doi:10.1103/PhysRevX.6.029902.

[5] I. Roth, R. Kueng, S. Kimmel, Y.-K. Liu, D. Gross, J. Eisert and M. Kliesch, Recovering quantum gates from few average gate fidelities, *Phys. Rev. Lett.* **121**, 170502 (2018). doi:10.1103/PhysRevLett.121.170502.