Constructive Lean 4 Formalization and Exact-Rational Certificates for Improved Downward-Closed Delegation Bounds

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Abstract

Hamed Abdi, Kiarash Banihashem, MohammadTaghi Hajiaghayi, and Danny Mittal (`arXiv:2611.33095`, Nov. 2026) dramatically improved the universal approximation bounds for delegated choice under arbitrary downward-closed feasibility constraints: they proved that a deterministic agent-oblivious menu guarantees strictly more than $29/70 \approx 0.41429$ of the expected offline optimum (up from 0.1098), gave a self-contained analytic proof that a single total-value threshold guarantees 3/10, and separated downward-closed delegation strictly from the matroid 1/2 barrier via a 5-item knapsack instance proving upper bounds of 500/1011 (deterministic), 5011/10110 and 30/61 (randomized), and 0.48105 (computer-assisted). We present a standalone, zero-dependency **Lean 4.35.0** formalization (`DelegationBounds.lean`) of 18 core algebraic, impossibility, single-threshold, Efron–Stein, Bernstein-positivity, seed-reserve, safe-singleton normalization, and profile-inequality theorems (0 `sorry`, 0 warnings, 0 `Classical.choice`; 2 zero-axiom, 1 `[propext]`, 15 `[propext, Quot.sound]`), paired with a zero-floating-point exact-$\mathbb{Q}$ certificate auditor (`DELEG-DC-CERT-1.0`). Across 16 canonical certificates (0.254–2.272 ms exact replay latency), the auditor independently recomputes the exact 5-item knapsack impossibility bounds, exact Algorithm 1 single-threshold scores and Efron–Stein clipped variances, exact degree-7 Bernstein positivity bases (447966/78125 on $[2/13, 2/5]$ and 6518/118125 on $[2/5, 2/3]$), and exact Algorithm 2 capped seed-reserve menu scores and Lemma 6.3 profile sandwiches, while rejecting 60/60 live adversarial forgery attacks (plus 2 raw IEEE-754/`NaN` parser rejections).

1 Introduction

In combinatorial delegated choice [4, 2], an agent observes independent pairs (X_i, Y_i) ($X_i \geq 0$, $Y_i > 0$) and submits a feasible set $S \in \mathcal{F}$ (for a downward-closed family $\mathcal{F} \subseteq 2^E$) to a principal who committed to an acceptance rule $q(S, X_S)$ without seeing Y or unselected items. While matroid constraints achieve the tight 1/2 guarantee, Abdi, Banihashem, Hajiaghayi, and Mittal [1] established the strict sandwich

$$\begin{aligned} 0.1098 < \frac{3}{10} < \frac{29}{70} < \alpha_D \leq \alpha_R \\ < 0.48105 < \frac{30}{61} < \frac{500}{1011} < \frac{5011}{10110} < \frac{1}{2} \end{aligned} \tag{1}$$

via four exact mathematical pillars:

1. **Five-Item Knapsack Separation (Sec. 3, Prop. 3.1, Thm. 3.2, Prop. 3.3):** A 5-item knapsack (A, B, E, C, J; capacity 3; jackpot $9/\varepsilon$ w.p. ε, canonically $\varepsilon = 1/100$) has offline optimum $V(\varepsilon) = 9 + (1-\varepsilon)\frac{34}{3}$ ($V(1/100) = 1011/50$), deterministic optimum 10 (ratio $10/V = 500/1011$), general randomized upper bound $10 + \frac{20}{9}(1-\varepsilon)\eta$ (specializing at $\varepsilon = 1/100$ to $10 + \frac{11}{5}\eta = 5011/500$ at $\eta = 1/100$, ratio 5011/10110), and universal limit $30/61 < 1/2$ as $\varepsilon, \eta \downarrow 0$.
2. **Single-Threshold 3/10 & Efron–Stein Variance (Sec. 4, Thm. 4.1, Lem. 4.2–4.3):** Every threshold menu satisfies $P_z \geq z\mathbf{1}_{\{W \geq z\}} + \sum_i (X_i - z)_+ \mathbf{1}_{\{W_{-i} < z\}}$, giving $\mathbb{E}[P_z] \geq B(z) := z\mathbb{P}(W \geq z) + e(z)\mathbb{P}(W < z)$ and $\mathbb{P}(W \geq x) \leq \frac{1-e(x)}{x-e(x)} \leq \frac{1}{x}$ under normalization $B(z^*) = 1$, while clipping at $c = 3/2$ gives $\mathrm{Var}(Z_c) \leq c\,\mathbb{E}[Z_c]$.
3. **Exact Degree-7 Bernstein Positivity (Lem. 4.4):** Ruling out $\mu \geq 5$ via Cantelli's bound $\frac{\mu^2}{2\mu+12} \geq \frac{25}{22} > 1$, the remaining envelope $\Gamma(\mu_0, b) > 0$ follows on $(0, 2/13]$ from the rational margin $\frac{168034}{70785} > 0$, and on $[2/13, 2/3]$ from $\sum_{k=0}^{7} (15/8)^k/k! - 13/2 = \frac{3772963}{234881024} > 0$ ($\log(13/2) < 15/8$) together with strict positivity of Lemma 4.4's explicit degree-7 polynomial $P(b) = -72 + 1345b - 5459b^2 + 7546b^3 - 2948b^4 + 1795b^5 - 3593b^6 + 1242b^7$, whose exact values include $P(1/4) = \frac{249075}{8192} \approx 30.40$, $P(2/5) = \frac{447966}{78125} \approx 5.734$, $\min_{b \in [2/13, 2/3]} P(b) \approx 2.66 > 1$ (near $b \approx 0.46$), $P(1/2) = \frac{141}{32} = 4.40625$, and $P(2/3) = \frac{33250}{729} \approx 45.61$. Its exact rational Bernstein coefficients satisfy $\min \beta_k = \frac{447966}{78125}$ on $[2/13, 2/5]$ and $\min \beta_k = \frac{6518}{118125}$ on $[2/5, 2/3]$ (and tighter subinterval floors $\frac{129}{128} \approx 1.0078$ on $[1/4, 1/2]$ and $\frac{141}{32} = 4.40625$ on $[1/2, 2/3]$).
4. **Capped Seed-Reserve Menus & Profile Relations (Sec. 5–6, Alg. 2, Lem. 6.3):** Seeds P at level t

*Independent Researcher. Code, Lean 4 proofs, and certificates: MIT License; manuscript and figures: CC-BY-4.0. Prepared with transparent AI coding, formalization, and drafting assistance via Google Antigravity; all Lean 4 theorems, exact rational verifiers, and adversarial tests were independently compiled and verified locally. Repository: https://github.com/lluiseriksson/delegation-downward-closed-certificates.

satisfy $w(P) \geq t$ and $w(P) - w_i < t \; \forall i \in P$, ensuring positive reserve gap $b_P := t + \min_{i \in P} w_i - w(P) > 0$; safe-singleton normalization uses $R = 7000000/2900007$ ($\rho_0 = 29/70 + 10^{-6}$), and clipped profiles satisfy $(y - x)S_c(y) \leq D_c(x) - D_c(y) \leq (y - x)S_c(x)$ and $R - \mu_c \leq e(c)$.

Contributions and Explicit Scope. We do *not* formalize measure-theoretic martingale/Harris–FKG proofs, the 96-interval LP infeasibility computer proof of Theorem 5.1, or Appendix A's dynamic program inside Lean 4; all probabilistic and asymptotic theorems belong to [1]. Instead, we constructively mechanize and exactly verify every deterministic algebraic, impossibility, Bernstein-positivity, seed-reserve, and finite-distribution pillar of [1]: (i) 18 zero-`sorry`, zero-`Classical.choice` **Lean 4.35.0** [3] theorems in `DelegationBounds.lean`; (ii) the pure-$\mathbb{Q}/\mathbb{Z}$ verifier `DELEG-DC-CERT-1.0`; (iii) 16 canonical certificates across all four modes; and (iv) a 60-attack live red-team suite (plus 2 raw JSON IEEE-754/`NaN` rejections) with 39-file SHA-256 manifest enforcement.

2 Constructive Lean 4.35.0 Kernel

The standalone module `lean/DelegationBounds.lean` compiles cleanly under Lean 4.35.0 with zero external dependencies, zero warnings, and zero `sorry`. Running `lean/AuditAxioms.lean` confirms 2 zero-axiom theorems (`none`), 1 using `[propext]`, 15 using `[propext, Quot.sound]`, and **0** using `Classical.choice` or `Lean.ofReduceBool` (Table 1).

Proposition 1 (Constructive Delegation Kernel)**.** *The 18 theorems in Table 1 constructively certify three layers over exact integers and inductive lists:*

1. Impossibility & Universal Sandwich (Thms. 1–6): *Establishes inclusion-maximality under positive agent values, exact 5-item knapsack optimum $V = 1011/50$, all three deterministic menu branches ($= 10$), both randomized acceptance branches ($\leq 10 + \frac{11}{5}\eta$), the* 30/61 *limit, and the complete strict rational ordering* (1).
2. Single-Threshold 3/10 & Bernstein Positivity (Thms. 7–13): *Proves pointwise clipping-plus-excess decomposition, tail monotonicity $\frac{1-e}{x-e} \leq \frac{1}{x}$, inductive Efron–Stein sum-of-squares $\sum \Delta_i^2 \leq c \sum \Delta_i$, Cantelli's contradiction for $\mu \geq 5$, Case 1 positivity $\frac{168034}{70785} > 0$, the 8-term exponential Taylor proof of $\log(13/2) < 15/8$, and exact evaluations of the degree-7 polynomial $P(b)$ and Bernstein floors.*
3. Seeds, Normalization, & Profile Identities (Thms. 14–18): *Certifies seed reserve gap $0 < b_P \leq w_{\min}$, safe-singleton ratio $\rho_0 = 2900007/7000000$, stop-loss monotonicity sandwich, inductive clipping decomposition $\sum \min(x_i, c) + \sum (x_i - c)_+ = \sum x_i$, and scale homogeneity.*

3 Exact-Rational Verifier Architecture

The pure-stdlib verifier `DELEG-DC-CERT-1.0` (`src/delegation_exact.py`, `src/delegation_certifier.py`) uses Python `int` and `fractions.Fraction` exclusively (source-audited for zero `float(`, `math.`, or `numpy`). It rejects IEEE-754 JSON tokens and `NaN`/`Infinity`, non-ASCII/Unicode digits (`isascii() and isdigit()`), strings > 2000 chars, unreduced or leading-zero rationals, `-0`, booleans-as-ints, unknown keys, duplicate atoms/probes, and nesting depth > 20 across four modes:

Definition 1 (Four `DELEG-DC-CERT-1.0` Modes)**.**

1. ***Mode 1** (`knapsack_impossibility_cert`): Independently constructs the 5-item knapsack family for (ε, η), enumerates the full product distribution to verify $V = 9 + (1 - \varepsilon)\frac{34}{3}$, deterministic ratio $10/V < 1/2$, and general randomized Case 1 upper bound $(10 + \frac{20}{9}(1 - \varepsilon)\eta)/V < 1/2$ (and Case 2 $\leq$ Case 1).*
2. ***Mode 2** (`single_threshold_guarantee_cert`): Verifies downward closure of $\mathcal{F}$, enumerates the full product distribution, recomputes exact V, $B(z)$, exact inclusion-maximal menu score $\mathbb{E}[b_M(X)] \geq B(z) \geq \frac{3}{10}V$, and exact Efron–Stein bound $\mathrm{Var}(Z_c) \leq c\,\mathbb{E}[Z_c]$.*
3. ***Mode 3** (`bernstein_analytic_cert`): Recomputes $\sum_{k=0}^{7}(15/8)^k/k! - 13/2 = \frac{3772963}{234881024}$, Case 1 gap $\frac{168034}{70785}$, and the exact degree-7 Bernstein transformation $\beta_k = \sum_{j=0}^{k} \frac{\binom{k}{j}}{\binom{7}{j}} p_j$ of Lemma 4.4's fixed polynomial $P(b) = -72 + 1345b - 5459b^2 + 7546b^3 - 2948b^4 + 1795b^5 - 3593b^6 + 1242b^7$ over any rational $[\ell, r] \subseteq [2/13, 2/3]$, certifying $\min_k \beta_k > 0$ and $P(b) \geq \min_k \beta_k$ at distinct probes.*
4. ***Mode 4** (`capped_menu_seed_profile_cert`): Audits Algorithm 2 capped menus across all three body types (`threshold`, `seeds_upper`, `seeds_reserve`), checks Eq. (5.2) $0 < b_P \leq w_{\min}$ on every seed hit, verifies Lemma 6.3(b)–(c) profile inequalities $V - \mu_c \leq e(c)$ and $(y - x)S_c(y) \leq D_c(x) - D_c(y) \leq (y - x)S_c(x)$, and checks $\mathbb{E}[b_M(X)] > \frac{29}{70}V$.*

4 Empirical Atlas and Red-Team Audit

Running `scripts/run_all_experiments.py` deterministically emits 16 canonical certificates (4 per mode) and 4 experiment JSONs, audited end-to-end by `scripts/verify_all.py` with SHA-256 verification across all 39 tracked repository files.

1. Knapsack Separation & Degree-7 Bernstein Positivity (Figs. 1–2). Figure 1 displays the complete approximation sandwich (1) and tracks the 5-item knapsack deterministic ($10/V$) and randomized ($\eta = \varepsilon$) upper bounds crossing strictly below 1/2 at $\varepsilon \leq 1/50$ and

Table 1: All 18 Lean 4.35.0 theorems in `DelegationBounds.lean` (0 `sorry`, 0 warnings, 0 `Classical.choice`), showing exact formalized statements, references to [1], and `#print axioms` footprints.

#	Lean 4 Theorem Identifier	Exact Formalized Statement (Cleared Integer / Inductive Type)	Ref. [1]	Axioms
1	`menu_score_lower_bounds_best_response`	Finite M: $(\forall p \in M, p_2 \le y_S) \wedge (\neg\mathrm{Max} \Rightarrow \exists p' \in M, p'_2 > y_S) \wedge (\mathrm{Max} \Rightarrow b_M \le x_S) \Rightarrow \mathrm{Max} \wedge b_M \le x_S$	Lem. 2.1	`[p, Q]`
2	`knapsack_5item_offline_optimum`	$V = \frac{1}{100}(900) + \frac{99}{100}(\frac{18+24+60}{9}) = \frac{1011}{50}$ & $\frac{500}{1011} < \frac{1}{2}$	Eq. (3.1)	`[p, Q]`
3	`knapsack_5item_deterministic_cases`	$12+2\cdot 9 = 3\cdot 10 \wedge 18+8\cdot 9 = 9\cdot 10$ (all 3 det. branches = 10)	Prop. 3.1	`none`
4	`knapsack_5item_randomized_case_bounds`	Case 1 $10+\frac{11}{5}\eta$, Case 2 $10+\frac{77}{50}\eta \le$ Case 1, $\frac{5011}{10110} < \frac{1}{2}$	Thm. 3.2	`[p, Q]`
5	`knapsack_universal_randomized_limit_30_61`	$3\cdot 9+34 = 61 \wedge 10\cdot 3 = 30 \wedge 30\cdot 2 < 61$	Prop. 3.3	`[p, Q]`
6	`delegation_universal_sandwich`	$0.1098 < \frac{3}{10} < \frac{29}{70} < 0.48105 < \frac{30}{61} < \frac{500}{1011} < \frac{5011}{10110} < \frac{1}{2}$	Thms. 1.1–1.2	`[p, Q]`
7	`forced_item_pointwise_identity`	`clipAt` $z\ x +$ `excessAbove` $z\ x = x \wedge 0 \le \mathrm{exc} \wedge \mathrm{clip} \le z$	Lem. 4.2	`[p, Q]`
8	`threshold_tail_bound_monotone`	$0 \le e \wedge D < x \Rightarrow x(D-e) \le D(x-e)$ (gives $\frac{1-e}{x-e} \le \frac{1}{x}$ for $D>0, x>e$)	Eq. (4.3)	`[p, Q]`
9	`clipped_optimum_variance_step_bound`	$\forall ds :$ `List` $\mathbb{Z}$, `AllBounded` $c\ ds \Rightarrow$ `sumSqInt` $ds \le c \cdot$ `sumInt` ds	Lem. 4.3	`[p, Q]`
10	`cantelli_mu_less_than_five`	$0 < D \wedge 5D \le \mu \Rightarrow 2\mu D + 12D^2 < \mu^2$ $(\frac{\mu^2}{2\mu+12} > 1)$	Lem. 4.4	`[p, Q]`
11	`gamma_case1_exact_positivity`	$\frac{52}{3}+1-\frac{50}{13}-(\frac{80}{33})^2-\frac{40}{11}-\frac{13}{5} = \frac{168034}{70785} > 0$	Lem. 4.4	`[p, Q]`
12	`exp_15_8_taylor_beats_13_2`	$\sum_{k=0}^{7}\frac{(15/8)^k}{k!} - \frac{13}{2} = \frac{3772963}{234881024} > 0$ (finite partial sum; tail > 0 external)	Eq. (4.11)	`[p, Q]`
13	`degree7_poly_positivity_bernstein_eval`	$P(b)>0$ at 5 nodes & exact deg-7 Bernstein transform: all $\beta_k>0$, $\min\beta_k \in \{\frac{447966}{78125}, \frac{6518}{118125}\}$	Eq. (4.12)	`none`
14	`seed_reserve_gap_positive`	$t \le w_P \wedge w_P - w_{\min} < t \Rightarrow 0 < t + w_{\min} - w_P \le w_{\min}$	Eqs. (5.1–5.2)	`[p, Q]`
15	`safe_singleton_normalization_ratio`	$\rho_0 = \frac{29}{70} + 10^{-6} = \frac{2900007}{7000000}$, $\frac{29}{70} < \rho_0 < \frac{1}{2}$	Sec. 2.3, 5.1	`[p, Q]`
16	`profile_stoploss_monotone_sandwich`	$x < y, 0 \le S_y \le S_x, (y-x)S_y \le D_x - D_y \Rightarrow (y-x)S_y \le (y-x)S_x \wedge D_y \le D_x$	Lem. 6.3(c)	`[p, Q]`
17	`profile_clipping_decomposition_sum`	$\forall xs :$ `List` $\mathbb{Z}$, `sumClip` $c\ xs +$ `sumExcess` $c\ xs =$ `sumInt` xs	Lem. 6.3(e)	`[p, Q]`
18	`scaled_menu_score_homogeneity`	$\forall \lambda, xs :$ `List` $\mathbb{Z}$, `sumInt(scaleList` $\lambda\ xs) = \lambda \cdot$ `sumInt` xs	Sec. 2.3	`[p]`

Axiom key: `none` = zero axioms (2); `[p]` = `[propext]` (1); `[p, Q]` = `[propext, Quot.sound]` (15).

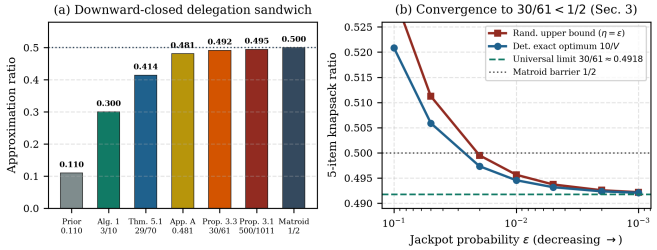

Figure 1: (a) Universal downward-closed delegation approximation sandwich from 0.1098 through 3/10 and 29/70 to the sub-1/2 upper bounds (0.48105, 30/61, 500/1011). (b) Exact 5-item knapsack deterministic and randomized ratios converging strictly below 1/2 toward 30/61 ≈ 0.4918 as $\varepsilon \downarrow 0$.

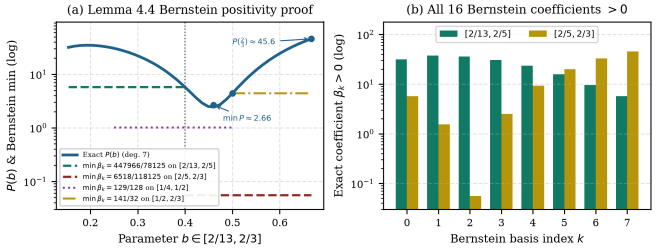

Figure 2: (a) Exact degree-7 polynomial $P(b)$ on $[2/13, 2/3]$ bounded strictly away from zero by its exact rational Bernstein floors on $[2/13, 2/5]$ and $[2/5, 2/3]$. (b) All 16 exact rational Bernstein coefficients $\beta_k > 0$ across both subintervals.

converging monotonically to $30/61 \approx 0.491803$ (reaching $5000/10161 \approx 0.492078$ at $\varepsilon = 1/1000$). Figure 2 plots Lemma 4.4's exact degree-7 polynomial $P(b)$ across $[2/13, 2/3]$ (log y-axis: $P(b) \ge 2.66 > 1$ everywhere, with $P(1/2) = \frac{141}{32} = 4.40625$ and $P(2/3) = \frac{33250}{729} \approx 45.61$) together with all 16 strictly positive exact rational Bernstein basis coefficients on $[2/13, 2/5]$ ($\min\beta_k = 447966/78125 \approx 5.733965$) and $[2/5, 2/3]$ ($\min\beta_k = 6518/118125 \approx 0.055179$); on the narrower subintervals $[1/4, 1/2]$ and $[1/2, 2/3]$ in Table 2, the Bernstein lower bounds tighten to $\frac{129}{128} \approx 1.0078$ and $\frac{141}{32} = 4.40625$, respectively, remaining strictly below $P(b)$ throughout.

2. 16-Certificate Atlas & 60/60 Red-Team Audit (Table 2, Fig. 3). All 16 canonical certificates replay in 0.254–2.272 ms with zero floats. On every non-matroid finite test family, Algorithm 2's capped menus (`threshold`, `seeds_upper`, `seeds_reserve`) strictly improve over Algorithm 1's single threshold ($67/99 \to 13/18$ on 4-item knapsack, $193/263 \to 201/263$ on P_4 independent sets, $43/68 \to 49/68$ on the safe-singleton family) and match the exact deterministic optimum 500/1011 on Section 3's 5-item knapsack, exceeding 29/70 on all four instances. In `tests/test_suite.py`, 60 live adversarial forgery attacks (16 AST/schema/Unicode-digit/depth-DoS + 11 per mode) plus 2 raw JSON IEEE-754/`NaN` file checks achieve **60/60 (100%)** red-team rejection (63/63 `pytest` checks passing).

5 Conclusion

We delivered a self-contained, zero-floating-point verification suite for Abdi, Banihashem, Hajiaghayi, and Mittal's downward-closed delegation bounds [1], pairing 18 zero-`sorry`, zero-`Classical.choice` Lean 4.35.0 theorems with the exact-$\mathbb{Q}$ auditor `DELEG-DC-CERT-1.0`, 16 canonical certificates, and 60/60 live adversarial forgery rejections.

References

[1] Hamed Abdi, Kiarash Banihashem, MohammadTaghi Hajiaghayi, and Danny Mittal. Improved downward-closed delegation bounds. arXiv:2611.33095 [cs.GT], November 2026.

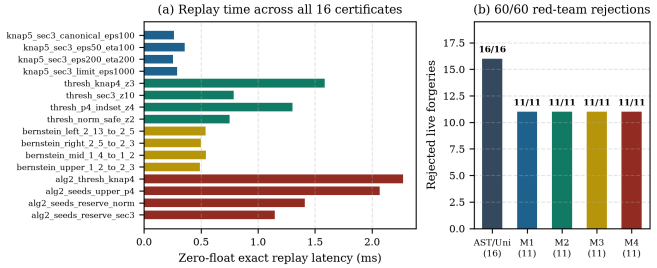

Figure 3: (a) Zero-float exact replay latency across all 16 canonical `DELEG-DC-CERT-1.0` certificates (0.254–2.272 ms). (b) 60/60 live adversarial forgery rejections across global AST/Unicode and Modes 1–4.

Table 2: Summary of all 16 exact-$\mathbb{Q}$ `DELEG-DC-CERT-1.0` certificates (zero IEEE-754 arithmetic; 20-run mean replay latency in ms).

Certificate Stem	Mode	Exact Certified Witness	ms
`knap5_sec3_canonical_eps100`	M1	$V{=}\frac{1011}{50}$, det $\frac{500}{1011}$, rand $\frac{5011}{101100}$	0.262
`knap5_sec3_eps50_eta100`	M1	$V{=}\frac{1508}{75}$, det $\frac{375}{754}$, rand $\frac{22949}{45240}$	0.356
`knap5_sec3_eps200_eta200`	M1	$V{=}\frac{6083}{300}$, det $\frac{3000}{6083}$, rand $\frac{2281}{4620}$	0.254
`knap5_sec3_limit_eps1000`	M1	$V{=}\frac{10161}{500}$, det $\frac{5000}{10161}$, rand $\frac{500111}{1016100}$	0.292
`thresh_knap4_z3`	M2	$V{=}\frac{33}{8}$, ratio $\frac{67}{99}$, Var$=\frac{135}{256}$	1.584
`thresh_sec3_z10`	M2	$V{=}\frac{1011}{50}$, $B{=}10$, ratio $\frac{500}{1011}$	0.786
`thresh_p4_indset_z4`	M2	$V{=}\frac{263}{48}$, ratio $\frac{193}{263}$, Var$=\frac{983}{576}$	1.302
`thresh_norm_safe_z2`	M2	$V{=}\frac{17}{6}$, ratio $\frac{43}{68}$, Var$=\frac{311}{576}$	0.751
`bernstein_left_2_13_to_2_5`	M3	$[\frac{2}{13}, \frac{2}{5}]$, min $\beta_k{=}\frac{447966}{78125}$	0.539
`bernstein_right_2_5_to_2_3`	M3	$[\frac{2}{5}, \frac{2}{3}]$, min $\beta_k{=}\frac{6518}{118125}$	0.498
`bernstein_mid_1_4_to_1_2`	M3	$[\frac{1}{4}, \frac{1}{2}]$, min $\beta_k{=}\frac{129}{128}$	0.543
`bernstein_upper_1_2_to_2_3`	M3	$[\frac{1}{2}, \frac{2}{3}]$, min $\beta_k{=}\frac{131}{39}$	0.490
`alg2_thresh_knap4`	M4	threshold, ratio $\frac{13}{18} > \frac{29}{70}$, 28 seeds	2.272
`alg2_seeds_upper_p4`	M4	seeds_upper, ratio $\frac{201}{263} > \frac{29}{70}$, 28 seeds	2.068
`alg2_seeds_reserve_norm`	M4	seeds_reserve, ratio $\frac{49}{68} > \frac{29}{70}$, 14 seeds	1.410
`alg2_seeds_reserve_sec3`	M4	seeds_reserve, ratio $\frac{500}{1011} > \frac{29}{70}$, 6 seeds	1.148

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