

Phase calibration unlocks adaptive tests for renewed quantum noise

Lluis Eriksson
Independent researcher

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Abstract

Can interventions distinguish an unknown phase that is frozen throughout an experiment from phase noise renewed at every use? We study ordinary forward queries, arbitrary quantum memory, and exactly zero false positives for every frozen diagonal unitary. For any Schur channel with correlation matrix C , the optimal parallel failure probability is $\min_{|m|=n} \text{per}(C[m])/n!$, where $C[m]$ repeats rows and columns according to the occupation vector m . The same formula is the exact optimum over all causal strategies when the renewed channel has an unknown common phase carrier and failure is evaluated in the worst case over that carrier. Calibration changes this conclusion: for qubit dephasing we give a rational five-query certificate that beats every parallel strategy for $0.5331424052 \dots < \eta < 1$. At $\eta = 0.7$, failure decreases from 0.46603 to 0.4602957928558. We prove physical realizability of the certificate, obtain exact causal limits at complete dephasing, and show that adaptation cannot improve the asymptotic failure exponent. The separation concerns finite-query performance under a calibrated statistical promise; it neither quantifies the hardware cost of a phase reference nor certifies a quantum environment.

1 Question, contribution, and scope

A random phase can be sampled once and retained, or sampled anew at each interaction. These mechanisms can have identical one-use channels and different multi-time behavior. The operational role of interventions and multi-time descriptions is well established [10]. Our question is more specific: how much can coherent control between uses improve *unambiguous exclusion of all frozen phases*?

There are two distinct promises for the renewed alternative. With a *calibrated carrier*, its channel is known. With an *uncalibrated carrier*, the channel is known only up to a fixed diagonal unitary applied at every slot, and a test must perform well for every such carrier. This distinction determines whether a finite-query adaptive advantage can occur. Theorem 4.1 rules out any advantage in the uncalibrated minimax problem, in every dimension and at every query count. Theorem 7.1 supplies a calibrated counterpoint using commuting qubit noise. Here and below, *adaptive* includes coherent intermediate controls and memory; it does not require a recorded intermediate measurement.

The permanent itself is not new to unambiguous comparison: Chefles, Andersson, and Jex obtained $\text{per}(\text{Gram})/N!$ as a symmetric-projector failure probability [1, Eq. (4.10)]. Our parallel theorem optimizes the input, including arbitrary ancillas, for a channel problem indexed by occupation types. Its extension to all causal tests under a common unknown carrier is the principal structural result. Likewise, causal Gram reconstruction is a specialization of established query methods, notably Block Sets [2, Definition 2, Theorems 2–3], semidefinite query characterizations [3], and quantum networks [4]. We provide the specialized construction because a numerical matrix inequality alone would not prove a physical adaptive separation.

Earlier unitary-comparison work [5, 6, 7] provides direct context, but uses different alternatives or access models. In particular, the updated multi-query comparison theorem of Hashimoto, Soeda, and Muraio [7] establishes parallel optimality for comparison of two Haar-unitary blocks, including general testers. Exact adaptive sensing limits for bosonic dephasing [8] and adaptive discrimination of specified channels [9] also predate this work. We claim neither a general invention of adaptive discrimination nor the first exact dephasing limit.

The author's earlier work concerns sampling static disorder without interventions within one realization [12], polynomial query support [13], and an exact two-query adaptive separation for tetrahedral echoes [14]. The present contribution is the all- d, n, C calibration dichotomy together with its certified qubit separation. Unlike the tetrahedral example, the calibrated causal optimum here remains open.

2 Model and notation

Fix $d \geq 2$ and $n \geq 1$. A query acts on a d -dimensional register in a known basis. Under the null, every slot applies the same unknown

$$D_\theta = \text{diag}(e^{i\theta_0}, \dots, e^{i\theta_{d-1}}), \quad \theta \in \mathbb{T}^d.$$

It may be sampled from any distribution once per run. The experiment outputs either rejection of the frozen-phase null or an inconclusive outcome. Its rejection probability must be exactly zero for *every* θ . We call the inconclusive probability under the alternative the failure probability β . This is a one-sided test; it does not unambiguously certify the null.

The calibrated alternative consists of independent uses of the Schur channel

$$\mathcal{E}_C(|a\rangle\langle b|) = C_{ab}|a\rangle\langle b|, \quad C \succeq 0, \quad C_{aa} = 1. \quad (1)$$

A dilation sends $|a\rangle$ to $|a\rangle|e_a\rangle$, with $\langle e_b, e_a \rangle = C_{ab}$, using a fresh environment at each slot. For a classical random-phase interpretation one additionally needs

$$C \in \text{conv}\{zz^\dagger : |z_a| = 1\}.$$

This extra condition holds for the qubit channels below, but does not follow from positive semidefiniteness and unit diagonal in general dimension $d \geq 4$ [11]. Our general theorems concern Schur channels; their classical phase-renewal interpretation has this restriction.

Allowed tests use ordinary forward queries, finite ancillary systems, ideal controls, and a final two-outcome measurement. Intermediate instruments and randomness may be purified and stored in memory. We do not allow inverse queries, controlled bypass of the queried device, environment access, or indefinite causal order. A parallel test prepares all n inputs jointly, queries them without intervening controls, and measures jointly. Write $\beta_{\text{par}}(n, C)$ and $\beta_{\text{causal}}(n, C)$ for the infima over parallel and all causal tests respectively.

A word $x = (x_1, \dots, x_n) \in \{0, \dots, d-1\}^n$ has occupation $m(x) = (m_0, \dots, m_{d-1})$. Let

$$\Omega_m = \{x : m(x) = m\}, \quad N_m = |\Omega_m| = \frac{n!}{\prod_a m_a!}, \quad |m| = n.$$

For a representative word $x \in \Omega_m$, let $C[m]$ be the $n \times n$ matrix with entries $C_{x_i x_j}$; its permanent is independent of the representative. Set

$$F_m(C) = \frac{\text{per}(C[m])}{n!}. \quad (2)$$

3 The exact parallel optimum

Theorem 3.1 (Optimized permanent law). *For every correlation matrix C , every d , and $n \geq 1$,*

$$\beta_{\text{par}}(n, C) = \min_{|m|=n} F_m(C). \quad (3)$$

An optimum uses no ancilla: prepare $|D_m\rangle = N_m^{-1/2} \sum_{x \in \Omega_m} |x\rangle$ for a minimizing type, and use the inconclusive effect $|D_m\rangle\langle D_m|$.

Proof. Purify an arbitrary parallel probe and write it as $|\psi\rangle = \sum_x |x\rangle|a_x\rangle$, with $p_x = \| |a_x\rangle \|^2$ and $\sum_x p_x = 1$. Its null output is

$$\sum_m e^{im\cdot\theta} |\psi_m\rangle, \quad |\psi_m\rangle = \sum_{x \in \Omega_m} |x\rangle|a_x\rangle.$$

Distinct characters are linearly independent, and the nonzero ψ_m are orthogonal. Thus the null span is their direct sum. If A is the inconclusive effect, zero rejection forces it to be identity on that span; since $0 \preceq A \preceq I$, it dominates the corresponding orthogonal projector.

Put $K = C^{\otimes n}$ and $p_m = \sum_{x \in \Omega_m} p_x$. The resulting lower bound on failure is

$$\sum_{m: p_m > 0} \frac{1}{p_m} \sum_{x, y \in \Omega_m} p_x p_y K_{xy}. \quad (4)$$

This expression retains arbitrary ancillas: the orthogonal word registers make the two inner products in each projector expectation equal to p_x and p_y . Each principal block $K_m = K|_{\Omega_m}$ is positive semidefinite. Consequently $q \mapsto q^\dagger K_m q$ is convex on the real probability simplex. Slot permutations preserve K_m and act transitively on Ω_m ; averaging $q_x = p_x/p_m$ over those permutations gives the uniform distribution without increasing the energy.

The uniform energy is the constant row sum of K_m divided by N_m . In the permanent of $C[m]$, each distinct word of type m occurs $\prod_a m_a!$ times. Hence this energy is

$$\frac{\text{per}(C[m])}{N_m \prod_a m_a!} = F_m(C).$$

Equation (4) is bounded below by $\sum_m p_m F_m(C)$, and thus by the smallest $F_m(C)$. The stated uniform-type probe and projector attain equality. $\square$

Remark 3.2. The argument includes complex and singular C . It does not assume entrywise nonnegativity. Its permanent is the familiar Gram-permanent quantity from state comparison [1]; the additional statement is global optimization over channel probes and ancillas.

4 Unknown carrier: a minimax theorem for all causal tests

Suppose the alternative is known only up to a fixed common carrier $\phi \in \mathbb{T}^d$:

$$C^{(\phi)} = D_\phi C D_\phi^\dagger, \quad \mathcal{E}_{C^{(\phi)}} = \mathcal{U}_\phi \circ \mathcal{E}_C, \quad \mathcal{U}_\phi(X) = D_\phi X D_\phi^\dagger.$$

Conditional on ϕ , the n uses are independent. The carrier is fixed throughout the run, rather than drawn anew at every slot. Define

$$\beta_{\text{causal}}^{\text{wc}}(n, C) = \inf_{T: \text{zero null error}} \sup_{\phi \in \mathbb{T}^d} \beta_T(C^{(\phi)}).$$

The infimum includes strategies that attempt to learn ϕ within their n queries.

Theorem 4.1 (Calibration-free minimax equality). *For every d, n, C as above,*

$$\beta_{\text{causal}}^{\text{wc}}(n, C) = \beta_{\text{par}}(n, C) = \min_{|m|=n} F_m(C). \quad (5)$$

Thus a uniform-type parallel test is minimax optimal among all causal tests.

Proof. A purified causal circuit has a phase-history expansion

$$|\Psi(z_1, \dots, z_n)\rangle = \sum_x \left(\prod_{j=1}^n z_{j,x_j} \right) |v_x\rangle, \quad |z_{j,a}| = 1. \quad (6)$$

Let $u_m = \sum_{x \in \Omega_m} v_x$. Under the frozen null the output is $\sum_m e^{im \cdot \theta} u_m$, of norm one for every θ . Haar integration and Parseval give

$$\sum_m \|u_m\|^2 = 1. \quad (7)$$

The vectors u_m need not be orthogonal.

Let A be the inconclusive effect. Zero rejection gives $A\Psi_\theta = \Psi_\theta$. Fourier extraction then implies $\sqrt{A}u_m = u_m$. Write $a_x = \sqrt{A}v_x$, so that $\sum_{x \in \Omega_m} a_x = u_m$. The alternative output, by the fresh-environment dilation of (1), has acceptance probability

$$\sum_{x,y} K_{xy} e^{i(m(x)-m(y)) \cdot \phi} \langle a_y, a_x \rangle.$$

Averaging ϕ deletes only cross-type terms. On one type block, slot-permutation invariance implies $K_m \mathbf{1} = N_m F_m(C) \mathbf{1}$. Decompose

$$a_x = \frac{u_m}{N_m} + b_x, \quad \sum_{x \in \Omega_m} b_x = 0.$$

The mixed terms vanish by the constant row sum, and positivity of K_m gives

$$\sum_{x,y \in \Omega_m} K_{xy} \langle a_y, a_x \rangle = F_m(C) \|u_m\|^2 + \sum_{x,y \in \Omega_m} K_{xy} \langle b_y, b_x \rangle \geq F_m(C) \|u_m\|^2. \quad (8)$$

For complex C , the last energy is nonnegative because it is the contraction of two positive Gram matrices (equivalently, a sum of squared tensor-product norms). Sum (8) and apply (7). The mean failure over ϕ , hence its supremum, is at least $\min_m F_m(C)$. No interchange of infimum and supremum is required. The uniform-type test in Theorem 3.1 has the same failure for every carrier, since $m(x) = m(y)$ throughout its support. It attains the lower bound. $\square$

Corollary 4.2 (Complete dephasing). *For $C = I_d$, even with a calibrated carrier,*

$$\beta_{\text{causal}}(n, I_d) = \beta_{\text{par}}(n, I_d) = \frac{1}{\max_{|m|=n} \binom{n}{m_0, \dots, m_{d-1}}}. \quad (9)$$

The maximum occurs when occupations differ by at most one.

Proof. The channel is invariant under carrier shifts, so its minimax and calibrated problems coincide. Here K_m is identity and $F_m = 1/N_m$. If $m_a \geq m_b + 2$, moving one count from a to b decreases $\prod_a m_a!$; iterating proves the balancing assertion. $\square$

5 Qubit formulas and the asymptotic limit

For qubits take a known zero carrier and

$$C_\eta = \begin{pmatrix} 1 & \eta \\ \eta & 1 \end{pmatrix}, \quad 0 \leq \eta \leq 1, \quad \mathcal{E}_\eta = p\text{Id} + q\mathcal{Z}, \quad p = \frac{1+\eta}{2}, \quad q = \frac{1-\eta}{2}, \quad (10)$$

where $\mathcal{Z}(X) = ZXZ$ and $Z = \text{diag}(1, -1)$. For a known nonzero carrier, a compensating control gives this model while preserving the composite frozen null.

Proposition 5.1 (Balanced qubit formula). *With $w = \lfloor n/2 \rfloor$,*

$$\beta_{\text{par}}(n, \eta) = \frac{\sum_{j=0}^w \binom{w}{j} \binom{n-w}{j} \eta^{2j}}{\binom{n}{w}}. \quad (11)$$

Proof. In a weight- w block, a fixed binary word has $\binom{w}{j} \binom{n-w}{j}$ words at Hamming distance $2j$, giving the displayed value for that weight. Let J_w have probabilities $\pi_w(j) = \binom{w}{j} \binom{n-w}{j} / \binom{n}{w}$. For $w < (n-1)/2$, the likelihood ratio on the common support is

$$\frac{\pi_{w+1}(j)}{\pi_w(j)} = \frac{(w+1)^2(n-w-j)}{(n-w)^2(w+1-j)},$$

which increases with j ; any new support lies at the upper end. Thus J_{w+1} stochastically dominates J_w . Since η^{2j} decreases in j , the block value decreases toward the balanced weight. Symmetry handles the other half. At $\eta = 0$ the value is $1/\binom{n}{w}$, and at $\eta = 1$ all values are one. $\square$

Proposition 5.2 (No improvement of the failure exponent). *For $n \geq 1$ and $0 \leq \eta \leq 1$,*

$$p^n + q^n \leq \beta_{\text{causal}}(n, \eta) \leq \beta_{\text{par}}(n, \eta) \leq (n+1)p^n. \quad (12)$$

Consequently, for fixed $0 \leq \eta < 1$, both optimal failure exponents are $-\log((1+\eta)/2)$.

Proof. The Kraus trajectories consisting entirely of I , or entirely of Z , have probabilities p^n and q^n . Each is a frozen null and must be inconclusive with certainty, for any causal protocol. This proves the lower bound. In (11), the numerator is at most $(1+\eta)^n$: it is a sub-sum of the product $(1+\eta)^w(1+\eta)^{n-w}$ with nonnegative terms. Also $\binom{n}{\lfloor n/2 \rfloor} \geq 2^n/(n+1)$. Taking logarithms and dividing by n proves the limit. $\square$

6 Causal certificates and physical realization

For completeness we specialize known query-decomposition methods [2, 3, 4] to the present continuous-phase setting. Index level k by words of length k , with the last symbol appended on the right. Let V be the $d^n \times \binom{n+d-1}{d-1}$ matrix whose columns are the type indicators $\mathbf{1}_{\Omega_m}$.

Lemma 6.1 (Causal Gram realization). *The following conditions characterize causal rejection statistics:*

$$Q_0 = [1], \quad Q_k \succeq 0, \quad (13)$$

$$(Q_k)_{xa,yb} = 0 \quad (a \neq b), \quad \sum_a (Q_k)_{xa,ya} = (Q_{k-1})_{x,y}, \quad (14)$$

$$0 \preceq R \preceq Q_n, \quad RV = 0. \quad (15)$$

The rejection probability under $\mathcal{E}_C^{\otimes n}$ is

$$s(C) = \text{Tr}(C^{\otimes n} R). \quad (16)$$

Every feasible collection has a realization using a d -dimensional query register and memory of dimension at most d^{n-1} .

Proof. Expand the purified output immediately after each query into its phase histories, with coefficient vectors $v_x^{(k)}$, and set $(Q_k)_{x,y} = \langle v_x^{(k)}, v_y^{(k)} \rangle$. Projection onto the last queried symbol makes distinct last-symbol branches orthogonal. Summing these projections preserves the previous inner products, proving (13)–(14). For a final rejection effect M , set $R_{x,y} = \langle v_x^{(n)}, M v_y^{(n)} \rangle$. Then $0 \preceq R \preceq Q_n$. Fourier extraction of the null characters shows that zero rejection is equivalent to annihilating all type sums; by positivity this is $RV = 0$. The channel dilation contracts two histories with $(C^{\otimes n})_{xy}$, giving (16), including for complex C .

Conversely, factor the same-last-symbol blocks of Q_k as $B_a^\dagger B_a$. If previous coefficient vectors have Gram Q_{k-1} , define on their span

$$v_x^{(k-1)} \mapsto \sum_a |a\rangle \otimes B_a |x\rangle.$$

The Gram recursion makes this map well defined and isometric, including when the previous vectors are linearly dependent. Extend the isometry to a unitary after completing the ambient spaces. Applying the next diagonal query produces exactly the required next coefficient vectors. This constructs all levels.

If S has the terminal vectors as columns, $S^\dagger S = Q_n$. The inequality $0 \preceq R \preceq S^\dagger S$ ensures that R annihilates $\ker S$. The effect $M = (S^+)^\dagger R S^+$ on $\text{ran } S$, extended by zero, satisfies $0 \preceq M \preceq I$ and $S^\dagger M S = R$. Each same-symbol block has size at most d^{k-1} . Embedding all levels in a space of dimension d^n proves the stated sufficient memory bound. $\square$

This lemma is an exact physical characterization, not a relaxation based only on polynomial normalization. It supplies a convenient specialization of the earlier reconstruction principles; we do not claim their general equivalence as new. The sufficient memory bound also says nothing about efficient compilation into a restricted gate set.

7 A certified calibrated separation

At $n = 5$, (11) becomes

$$\beta_{\text{par}}(5, \eta) = \frac{1 + 6\eta^2 + 3\eta^4}{10}. \quad (17)$$

The supplementary files `phase-certificate-Q.json` and `phase-certificate-R.json` specify rational 32×32 matrices, in binary lexicographic order. They define a single fixed strategy, independent of η .

Theorem 7.1 (Five-query calibrated advantage). *There is a causal five-query test with exactly zero rejection of every frozen phase, whose rejection probability is*

$$\begin{aligned} s_*(\eta) = & \frac{1086981}{1250000} + \frac{3225799}{50000000}\eta - \frac{3784393}{6250000}\eta^2 \\ & - \frac{585391}{25000000}\eta^3 - \frac{103157}{390625}\eta^4 - \frac{2055017}{50000000}\eta^5. \end{aligned} \quad (18)$$

It strictly improves on every parallel test for $\eta_* < \eta < 1$, where η_* is the unique real root of

$$g(\eta) = 2055017\eta^3 - 1795904\eta^2 + 3225799\eta - 1520760$$

and $0.53314 < \eta_* < 0.53315$. At $\eta = 7/10$, its failure and improvement are respectively

$$1 - s_*(7/10) = \frac{2301478964279}{5000000000000}, \quad \beta_{\text{par}}(5, 7/10) - (1 - s_*(7/10)) = \frac{28671035721}{5000000000000}. \quad (19)$$

Proof with a finite rational certificate. Reduce $Q = Q_5$ recursively by $(Q_{k-1})_{xy} = (Q_k)_{x0,y0} + (Q_k)_{x1,y1}$. The supplied entries satisfy all off-last-symbol zeros and $Q_0 = [1]$. Exact rational Schur-complement elimination establishes

$$Q \succ 0, \quad Q - R \succ 0, \quad R \succeq 0, \quad \text{rank } Q = \text{rank}(Q - R) = 32, \quad \text{rank } R = 26.$$

The six type-indicator columns are annihilated exactly by R . Lemma 6.1 therefore supplies a physical test with one query qubit and at most a 16-dimensional memory.

Summing R_{xy} over each Hamming distance gives the six coefficients in (18). Subtracting its failure from (17) gives the exact identity

$$\beta_{\text{par}}(5, \eta) - (1 - s_*(\eta)) = \frac{(1 - \eta^2)g(\eta)}{50000000}. \quad (20)$$

The derivative of g has positive leading coefficient and negative discriminant -66647776694132 , so g increases on the real line. Exact substitution at 0.53314 and 0.53315 brackets its root. Equations (20) and (19) follow.

The included verifier replays all checks in exact rational arithmetic. No semidefinite optimizer is needed. Positive semidefiniteness is checked by successive Schur complements: a negative pivot fails, and a zero pivot is allowed only if its remaining row is identically zero. This yields an exact finite-arithmetic proof for the supplied entries. $\square$

	Test at $\eta = 0.7$	Failure	Detection
Optimal parallel	0.4660300000000	0.5339700000000	
Certified causal	0.4602957928558	0.5397042071442	

The improvement is about 0.5734 percentage points of detection. It is mathematically strict but modest. The theorem does not identify the optimal causal test, prove that five queries are necessary, or improve the exponent in Proposition 5.2. In particular, a floating-point optimizer value is not an additional exact theorem. At $\eta = 0$, the exact optimum is $1/\binom{5}{2} = 0.1$ by Corollary 4.2; the displayed certificate is not optimal there. At $\eta = 1$, the alternative is itself a frozen unitary.

8 Equal marginals and perturbations

Proposition 8.1 (Matched one-use channels). *For $0 \leq \eta < 1$, requiring zero rejection against every frozen qubit phase law with first moment η is equivalent to requiring zero rejection for each deterministic frozen phase.*

Proof. Fix a point z on the unit circle. Choose $0 < \epsilon \leq (1 - \eta)/2$. The residual mean $(\eta - \epsilon z)/(1 - \epsilon)$ lies in the unit disk, because $|\eta - \epsilon z| \leq \eta + \epsilon \leq 1 - \epsilon$. It is a convex combination of two circle points. Thus a phase law of mean η can assign positive mass ϵ to any prescribed z . Zero rejection averaged over every such law, together with nonnegativity, forces zero rejection at that point. The converse is immediate. $\square$

In particular, sampling I or Z once with probabilities p, q and retaining it has the same one-use channel as independently sampling them at every slot. The proposition concerns *all* compatible frozen laws, rather than a known two-point null law alone. Rejection excludes the specified frozen-phase null within the promise; it does not establish universal Markovianity, an intrinsically quantum bath, or a unique physical noise mechanism.

Proposition 8.2 (Qubit null perturbations). *For any zero-null-error qubit test, suppose the relative phase at slot j is $\theta + \delta_j$, with $|\delta_j| \leq \pi$. Its false-positive probability is at most*

$$\min \left\{ 1, \left(\frac{1}{2} \sum_{j=1}^n |\delta_j| \right)^2 \right\}. \quad (21)$$

A common phase change across all slots remains an exact null.

Proof. After removing a global scalar, the operator-norm change of one qubit phase query is $2|\sin(\delta_j/4)| \leq |\delta_j|/2$. Telescoping the purified isometric circuit bounds the final vector change by the sum. The square root of the rejection effect annihilates the ideal null vector and is a contraction. Squaring gives (21). $\square$

This is a bound on a fixed test’s false positives. It does not prove superiority to the best parallel test allowed a positive false-positive level. Translating time errors into δ_j requires a frequency or phase-width bound. The result is not a claim of clock robustness without a physical scale, and it does not include arbitrary state preparation, measurement, or memory errors.

9 Reproducibility and remaining questions

The certificate matrices were obtained by numerical semidefinite search, interior mixing, rational rounding, and exact constraint repair. The final proof depends on the rational entries and their verifier, not on optimizer tolerances. The two matrix files have SHA-256 hashes

Q: 18ecfa37bc84e3b0ca40efdc9dbdda6b87442479d76a17f4f7ff04563f088463
R: 8a3bc80cdc93463652359b7145a095ac528c1469486173fb3f3cc3ad8e4000be

The standard-library verifier checks causal constraints, positivity, null annihilation, coefficient aggregation, and the exact separation. A separate numerical reconstruction checks the circuit against frozen phases, independent phase histories, and direct Kraus evolution; its largest probability discrepancy was below 10^{-14} . These numerical checks are diagnostics, not proofs of continuum claims.

A separate GPT-6 Astra review context checked the candidate proofs, reimplemented rational verification, reconstructed the circuit, and tested the minimax argument. That review shares the model provider and was coordinated with the authoring process; it is neither blind review nor human peer review. Its report and scope are included with the supplement. AI systems assisted literature triage, mathematical development, coding, drafting, and review. No formal proof-assistant verification is claimed.

The literature search renewed indexes from frontier-AI mathematics collections and the author’s repositories, followed by targeted reading of the closest quantum-information sources. Index coverage is not full-text mathematical verification. The accompanying search audit records that distinction; the absence of a located identical result is not a guarantee of priority.

Three questions remain especially concrete. First, what is the exact calibrated causal optimum, and what is the smallest query count admitting a strict advantage? Second, can the five-query certificate be replaced by a low-memory analytic circuit that explains the beneficial interference between occupation sectors? Third, does a useful advantage persist after jointly optimizing both test classes at a positive false-positive allowance and with realistic control costs? The present results settle the uncalibrated minimax problem and exhibit a finite calibrated separation, while leaving these stronger claims open.

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