Machine-Checked Exact-Rational Koashi–Imoto Decomposition, Qubit Dichotomy, and Pure-Atom Certificates for Holevo-Rate Visible Compression

Lluis Eriksson*

November 25, 2026

Abstract

Polloreno (`arXiv:2611.33129`, November 24, 2026) resolved when visible compression of a finite memoryless quantum source $\{p_x, \omega_x\}$ with shared classical randomness achieves the Holevo rate $R_{\mathrm{cr}} = \chi$: in the maximal Koashi–Imoto (KI) decomposition $\omega_x = \bigoplus_j p(j|x)\, \rho_x^j \otimes \xi_j$, achievability holds if and only if every occupied quantum core ρ_x^j is pure—reducing for qubits to the dichotomy that all signals are pure or commute pairwise. We present a machine-checked, zero-floating-point formalization and certificate auditing framework for the finite-dimensional algebraic core of this criterion. First, we build an 18-theorem zero-dependency Lean 4 kernel (`KoashiImotoHolevo.lean`, Lean 4.35.0; 0 `sorry`, 0 warnings, 0 `Classical.choice`) certifying exact Bloch purity and linear-entropy defects, cross-product commutator antisymmetry/collinearity, the 3D Lagrange identity $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$, strict convex purity drop along off-center chords, 2×2 Cayley–Hamilton rank-1 idempotence and commutator criteria, KI Kronecker trace and purity multiplicativity, the visible/blind rate sandwich and separation ($R_{\mathrm{cr}} = \chi < R_{\mathrm{blind}}$), decomposition-cost excess, Proposition 9 partial additivity, inductive Proposition 10 sector-splitting excess and all-cores-zero biconditional rigidity, and Lemma 3 Carathéodory positivity $|\mathcal{X}|d^2 + 1 \geq 2$. Second, we define `KOASHI-IMOTO-CERT-1.0` and a pure-stdlib zero-float verifier (using exact rational LDL^T PSD checks and Wedderburn/Schur trivial-commutant Gaussian elimination) spanning four modes: qubit Bloch dichotomies, multi-sector block-tensor KI decompositions (up to 6×6), pure-atom Carathéodory mixtures without square roots, and sector-splitting / partial-additivity audits. Across 16 canonical certificates, zero-float replay executes in 0.075–0.655 ms, passes independent exact-$\mathbb{Q}[i]$ Hermitian and PSD cross-checks, and rejects 57/57 live adversarial forgeries (including reducible 3×3 direct sums, hidden 4×4 redundant factors, and partially commuting mixed cores).

1 Introduction

In visible quantum source compression, an encoder who knows the classical source label x transmits a quantum message to a decoder using unlimited shared classical randomness (and no shared entanglement) to reproduce the ensemble $\{p_x, \omega_x\}_{x \in \mathcal{X}}$ on a finite-dimensional register W [6]. While the optimal rate R_{cr} is always bounded below by the Holevo information $\chi = S(\bar{\omega}) - \sum_x p_x S(\omega_x)$, it remained open whether shared randomness suffices to attain $R_{\mathrm{cr}} = \chi$ for arbitrary mixed ensembles. Polloreno [5] settled this question via the maximal Koashi–Imoto (KI) decomposition [4, 3]: $R_{\mathrm{cr}} = \chi$ if and only if every occupied quantum core ρ_x^j is pure, whereas any mixed occupied quantum core induces a uniform rate gap $R_{\mathrm{cr}} > \chi$ (proved via Fawzi–Renner recovery [2] and blind-compression decoupling).

Crucially, checking the KI criterion numerically in floating-point arithmetic is brittle: a tiny numerical perturbation can falsely declare a rank-1 core ρ_x^j mixed ($\mathrm{Tr}((\rho_x^j)^2) = 1 - 10^{-15}$), overlook a hidden block diagonalization where a supposedly mixed sector actually commutes pairwise, or accept a non-positive-semidefinite (indefinite) forged density matrix. Because purity ($\rho^2 = \rho$), pairwise commutativity ($[\rho_x, \rho_y] = 0$), exact block-tensor factoriza-

*Independent researcher; manuscript prepared with transparent AI assistance via Google Antigravity. Source code, Lean 4 proofs, and exact certificates: https://github.com/lluiseriksson/koashi-imoto-holevo-certificates.

tion $\bigoplus_j p(j|x)(\rho_x^j \otimes \xi_j)$, and exact positive semidefiniteness (via rational LDL^T pivoting) are purely algebraic over $\mathbb{Q}$, they admit zero-floating-point, machine-checkable certificates.

Contributions. We formally verify and audit the finite-dimensional algebraic structure of [5], explicitly scoping our claims to exact finite-dimensional algebraic invariants, rational density-matrix witnesses, and cleared linear entropy/rate identities rather than the asymptotic Fawzi–Renner converse and regularized multi-copy entropy limits of [5]:

1. **Zero-dependency Lean 4 kernel (§3):** 18 constructive theorems in `KoashiImotoHolevo.lean` (Lean 4.35.0 [1]), 0 `sorry`, 0 warnings, 0 `Classical.choice`.
2. **Exact-$\mathbb{Q}$ verifier and schema (§4):** `KOASHI-IMOTO-CERT-1.0`, audited purely via Python's `fractions.Fraction` and arbitrary-precision `int` (zero floats, exact rational LDL^T PSD test) across four modes.
3. **Empirical atlas, Hermitian cross-checks, and red-team audit (§5):** 16 canonical certificates—including Polloreno's Fig. 2(a,b,c) qubit regimes, the exact qutrit erasure separation $R_{\mathrm{cr}} = \chi < R_{\mathrm{blind}}$, and 4D/6D multi-sector KI decompositions—replayed in 0.075–0.655 ms, plus 57/57 rejected adversarial forgeries.

2 Algebraic Koashi–Imoto Criterion

Let $\{p_x, \omega_x\}_{x\in\mathcal{X}}$ ($p_x > 0$, $\sum_x p_x = 1$) be a finite quantum ensemble on $\mathcal{H}_W$ ($\dim W = d$). In a Koashi–Imoto decomposition $\mathcal{H}_W \cong \bigoplus_{j=1}^{J}(\mathcal{H}_{Q_j} \otimes \mathcal{H}_{N_j})$ (Eq. (3) of [5]), every signal takes the exact block-tensor form

$$\omega_x = \bigoplus_{j=1}^{J} p(j|x)\, \rho_x^j \otimes \xi_j, \tag{1}$$

where ξ_j is strictly *independent of* x and $\sum_j p(j|x) = 1$. Writing $r_{\mathrm{mix}} = S(Q|XJ) = \sum_{x,j} p_x p(j|x) S(\rho_x^j)$, Theorem 1 of [5] establishes the sandwich $\chi \le R_{\mathrm{cr}} \le \chi + r_{\mathrm{mix}}$ with $R_{\mathrm{cr}} = \chi \iff r_{\mathrm{mix}} = 0$, whereas blind compression obeys $R_{\mathrm{blind}} - \chi = H(J|X) + r_{\mathrm{mix}}$. Over $\mathbb{Q}$, exact core purity $S(\rho_x^j) = 0$ is algebraically equivalent to idempotence $(\rho_x^j)^2 = \rho_x^j$ and vanishing linear-entropy mixing defect

$$r_{\mathrm{lin}} = \sum_{x,j} p_x p(j|x) \left(1 - \mathrm{Tr}\big((\rho_x^j)^2\big)\right) \in \mathbb{Q}_{\ge 0}, \tag{2}$$

so $r_{\mathrm{lin}} = 0 \iff r_{\mathrm{mix}} = 0$. For a qubit ensemble $\omega_x = \frac{1}{2}(I + \vec{r}_x \cdot \vec{\sigma})$ with rational Bloch vectors $\vec{r}_x \in \mathbb{Q}^3$ ($\|\vec{r}_x\|_2^2 \le 1$), purity is $\mathrm{Tr}(\omega_x^2) = \frac{1+\|\vec{r}_x\|_2^2}{2}$ and $[\omega_x, \omega_y] = \frac{i}{2}(\vec{r}_x \times \vec{r}_y) \cdot \vec{\sigma}$, yielding the exact algebraic qubit dichotomy:

$$\begin{aligned} R_{\mathrm{cr}} = \chi \iff & \big(\forall x,\ \|\vec{r}_x\|_2^2 = 1\big) \\ & \vee\ \big(\forall x, y,\ \vec{r}_x \times \vec{r}_y = \vec{0}\big). \end{aligned} \tag{3}$$

Furthermore, by Lemma 3 of [5], the decomposition cost $F(\sigma) = \inf_U [I(X:U) + S(W|U)]$ is attained by pure conditional atoms $\omega_x = \sum_u p(u|x)\phi_{xu}$ with $|U| \le |\mathcal{X}|d^2 + 1$; for any nonzero rational state vector $|\psi_{xu}\rangle \in \mathbb{Q}^d$, the rank-1 projector $\phi_{xu} = \frac{|\psi_{xu}\rangle\langle\psi_{xu}|}{\langle\psi_{xu}|\psi_{xu}\rangle} \in \mathbb{Q}^{d\times d}$ is exact over $\mathbb{Q}$ with zero square roots.

3 Zero-Dependency Lean 4 Kernel

We formalize 18 constructive theorems in `lean/KoashiImotoHolevo.lean` (Lean 4.35.0, zero external libraries), audited via `lean/AuditAxioms.lean`. All 18 compile cleanly with 0 warnings, 0 `sorry`, 0 `Classical.choice`, and 0 `Lean.ofReduceBool` (all 18 depend solely on `[propext, Quot.sound]`). Table 1 enumerates the complete theorem suite.

Theorem 1 (Constructive Koashi–Imoto & Compression Kernel)**.**

1. Bloch geometry & off-center chords: *Theorems 1–7 prove exact cleared purity equivalence* $S^2 + |\vec{r}|^2 = 2S^2 \iff |\vec{r}|^2 = S^2$, *strict mixed defect, cross-product antisymmetry, self-commutativity, collinearity commutativity, the 3D Lagrange identity* $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$, *and strict convex purity drop* $|p\vec{u} + q\vec{v}|^2 < (p+q)^2 S^2$ *along non-degenerate chords* ($p, q > 0$, $\vec{u} \ne \vec{v}$).
2. Matrix idempotence & Kronecker blocks: *Theorems 8–11 prove* 2×2 *Cayley–Hamilton rank-1 idempotence* $M^2 = SM$ *(*$\mathrm{Tr} = S, \det = 0$*), the* 2×2 *commutator criterion, and exact KI tensor-factor trace and purity multiplicativity* $\mathrm{Tr}((\rho \otimes \xi)^2) = \mathrm{Tr}(\rho^2)\mathrm{Tr}(\xi^2)$.
3. Rate bounds, sector splitting, & Carathéodory: *Theorems 12–18 formalize Theorem 1 equality* $R_{\mathrm{cr}} = \chi$ *at* $r_{\mathrm{mix}} = 0$, *the blind-vs-visible separation* $R_{\mathrm{cr}} = \chi < R_{\mathrm{blind}}$ *when* $H(J|X) > 0$, *Eq. (12)*

Table 1: Complete 18-theorem Lean 4.35.0 kernel in `KoashiImotoHolevo.lean` (0 `sorry`, 0 warnings, 0 `Classical.choice`).

#	Lean 4 Theorem Identifier	Axiom Footprint
1	`bloch_purity_pure_iff_unit_norm`	[propext, Quot.sound]
2	`bloch_purity_strict_mixed_defect`	[propext, Quot.sound]
3	`bloch_cross_antisymm`	[propext, Quot.sound]
4	`bloch_cross_self_zero`	[propext, Quot.sound]
5	`bloch_collinear_commute`	[propext, Quot.sound]
6	`lagrange_cross_dot_identity`	[propext, Quot.sound]
7	`bloch_chord_convex_purity_drop`	[propext, Quot.sound]
8	`qubit_2x2_trace_idempotent_rank1`	[propext, Quot.sound]
9	`qubit_2x2_commutator_zero_iff_collin`	[propext, Quot.sound]
10	`ki_kronecker_trace_multiplicative`	[propext, Quot.sound]
11	`ki_kronecker_purity_multiplicative`	[propext, Quot.sound]
12	`holevo_visible_sandwich_equality`	[propext, Quot.sound]
13	`blind_vs_visible_gap_decomposition`	[propext, Quot.sound]
14	`decomposition_cost_identity_excess`	[propext, Quot.sound]
15	`partial_additivity_pure_core_exact`	[propext, Quot.sound]
16	`sector_splitting_excess_list`	[propext, Quot.sound]
17	`sector_excess_zero_iff_all_cores_zero`	[propext, Quot.sound]
18	`caratheodory_pure_atom_bound_pos`	[propext, Quot.sound]

excess, Proposition 9 exact additivity for pure-core factors, inductive Proposition 10 sector-splitting excess and all-cores-zero rigidity over `List`, and Lemma 3 Carathéodory positivity $|\mathcal{X}|d^2 + 1 \geq 2$.

4 Exact-Rational Certificate Architecture

The pure-stdlib verifier `src/ki_certifier.py` audits `KOASHI-IMOTO-CERT-1.0` certificates with zero floating-point arithmetic, strict JSON AST type enforcement (rejecting `float`, `bool`, `Decimal`, `complex`, tuples, non-string keys, unreduced fractions, denominator-1 strings, and `"-0"`), and an exact symmetric-pivoting rational LDL^T positive-semidefiniteness checker (`mat_is_psd_exact`) for every real-symmetric density matrix.

Definition 1 (Four `KOASHI-IMOTO-CERT-1.0` Audit Modes)**.**

1. ***Mode 1*** *(`qubit_bloch_dichotomy_exact`): Audits positive priors* ($\sum p_x = 1$) *and rational Bloch vectors* $\|\vec{r}_x\|_2^2 \leq 1$*, recomputing exact* r_{lin}*,* $\max_{x,y} \|\vec{r}_x \times \vec{r}_y\|_2^2$*, and exact dichotomy classification* (3)*.*
2. ***Mode 2*** *(`ki_block_tensor_criterion_exact`): Checks PSD/unit-trace on all* $\omega_x, \rho_x^j, \xi_j$ ($d_W \in [2, 8]$)*, bit-exact direct-sum Kronecker reconstruction* (1)*, exact core idempotence* $(\rho_x^j)^2 = \rho_x^j$ *and* r_{lin}*, and—whenever claiming `strict_gap_mixed_core`—verifies exact trivial commutant* $\dim_{\mathbb{Q}}\{X : [X, \rho_x^j] = 0 \ \forall x\} = 1$ *via rational Gaussian elimination for* every *mixed occupied sector (ruling out reducible direct sums, hidden redundant factors, and partially commuting sectors!).*
3. ***Mode 3*** *(`pure_atom_caratheodory_decomp_exact`): Audits pure-atom convex reconstructions* $\omega_x = \sum_u p(u|x)\phi_{xu}$ *from rational state vectors* $|\psi_{xu}\rangle \in \mathbb{Q}^{d_W}$ *(zero square roots), exact projector idempotence* $\phi_{xu}^2 = \phi_{xu}$*, and Lemma 3 cardinality* $1 \leq |U| \leq |\mathcal{X}|d_W^2 + 1$*.*
4. ***Mode 4*** *(`sector_splitting_and_partial_additivity_exact`): Audits exact algebraic consistency of externally supplied rational rate/entropy witness tuples against Proposition 10 sector-splitting totals* (F, χ*, visible excess* $\sum q_j(F_j - \chi_j) \leq r_{\text{mix}}$*, blind excess* $H(J|X) + r_{\text{mix}}$*) and the Proposition 9 partial-additivity sandwich* $\chi_1 + F_2 \leq F_{12} \leq F_1 + F_2$ *(enforcing exact additivity* $F_{12} = F_1 + F_2$ *whenever* $F_1 = \chi_1$*).*

5 Empirical Verification and Red-Team Results

All 16 canonical certificates and four experiments are produced deterministically by `scripts/run_all_experiments.py` and verified end-to-end by `scripts/verify_all.py` (Lean 4 build, 18/18 axiom audit, 16/16 zero-float cert replays, `pytest`, and 39/39 SHA-256 manifest hashes).

1. Off-center qubit chord rates and exact purity defect. Figure 1(a)–(b) sweeps the mixing parameter $t_0 \in \{0, \frac{1}{12}, \frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \frac{5}{12}, \frac{1}{2}\}$ along the exact rational off-center Bloch chord joining pure endpoints $\vec{u} = (\frac{3}{5}, 0, \frac{4}{5})$ and $\vec{v} = (-\frac{3}{5}, 0, \frac{4}{5})$ (exact projectors from $|\psi_\pm\rangle = (3, \pm 1)^T$). At $t_0 = 0$ (pure non-orthogonal), $r_{\text{lin}} = 0$ and Schumacher compression attains $\chi = 0.468996$ qubits; at $t_0 = 1/2$ (identical commuting states), $\|\vec{r}_0 \times \vec{r}_1\|_2^2 = 0$ and $\chi = 0$. Everywhere in the open interior $0 < t_0 < 1/2$—such as our certified point $t_0 = 1/6$ with exact $r_{\text{lin}} = 1/10$ and $\|\vec{r}_0 \times \vec{r}_1\|_2^2 = 256/625$—both pure endpoints and pairwise commutativity fail simultaneously, yielding a strict gap between $\chi = 0.170878$ and $\min(C, S(\bar{\omega})) = 0.349978$ (upper gap 0.179099 qubits).

2. Qutrit erasure separation and Carathéodory scaling. Figure 2(a) il-

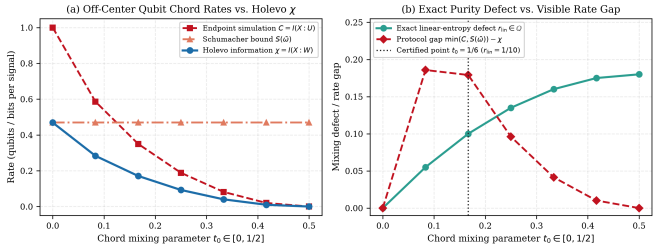


Figure 1: (a) Off-center qubit chord compression rates: endpoint simulation $C = I(X : U)$, Schumacher bound $S(\bar{\omega})$, and Holevo information χ. (b) Exact rational linear-entropy mixing defect $r_{\text{lin}} \in \mathbb{Q}$ versus protocol gap $\min(C, S(\bar{\omega})) - \chi$.

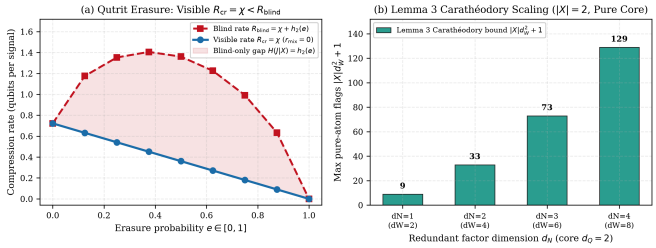


Figure 2: (a) Qutrit erasure separation: visible compression attains $R_{\text{cr}} = \chi$ across all $e \in [0, 1]$, while blind compression suffers gap $H(J|X) = h_2(e)$. (b) Lemma 3 Carathéodory pure-atom flag ceiling $|X|d_W^2 + 1$ versus redundant dimension d_N.

lustrates Polloreno's qutrit erasure source $\omega_x = e|e\rangle\langle e| \oplus (1-e)|\psi_x\rangle\langle\psi_x|$ ($|\langle\psi_0|\psi_1\rangle| = 3/5$, pure occupied cores in both sectors): because $r_{\text{mix}} = 0$, visible compression with shared randomness tracks $\chi(e) = (1-e)h_2(4/5)$ identically ($R_{\text{cr}} = \chi$, zero gap), whereas unassisted blind compression pays the full sector-uncertainty penalty $R_{\text{blind}} - \chi = H(J|X) = h_2(e)$, peaking at 1.0 qubit excess at $e = 1/2$ ($\chi = 0.360964$ vs. $R_{\text{blind}} = 1.360964$). Figure 2(b) tracks the Lemma 3 Carathéodory pure-atom bound $|X|d_W^2 + 1 \in \{9, 33, 73, 129\}$ as redundant factor dimension d_N grows from 1 to 4 while the pure quantum core keeps $r_{\text{lin}} = 0$ identically.

3. 16-certificate atlas, Hermitian cross-checks, and 57-attack red-team suite. Table 2 and Figure 3 summarize all 16 canonical certificates across all four modes, replayed with zero floating-point arithmetic in 0.075–0.655 ms (including 6×6 two-sector mixed-core KI reconstruction and 4×4 pure-core $\otimes$ mixed-redundant states

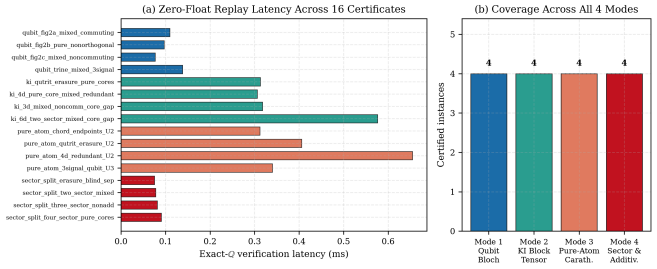


Figure 3: (a) Zero-float replay latency across all 16 canonical `KOASHI-IMOTO-CERT-1.0` certificates (0.075–0.655 ms). (b) Balanced 4-per-mode coverage across all four verification modes.

Table 2: Summary of all 16 exact-$\mathbb{Q}$ `KOASHI-IMOTO-CERT-1.0` certificates (zero floating-point arithmetic; reference-machine replay latency in ms).

Certificate File (stem)	Mode	Key Exact Witness	ms
`qubit_fig2a_mixed_commuting`	M1	commuting, $r_{\text{lin}} = 51/200$, $\|\times\|^2 = 0$	0.110
`qubit_fig2b_pure_nonorthogonal`	M1	pure, $r_{\text{lin}} = 0$, $\|\times\|^2 = 576/625$	0.097
`qubit_fig2c_mixed_noncommuting`	M1	gap, $r_{\text{lin}} = 1/10$, $\|\times\|^2 = 256/625$	0.077
`qubit_trine_mixed_3signal`	M1	gap, $r_{\text{lin}} = 11/50$, $\|\times\|^2 = 144/625$	0.138
`ki_qutrit_erasure_pure_cores`	M2	$d_W = 3$, $J = 2$, $r_{\text{lin}}^{\text{core}} = 0$	0.313
`ki_4d_pure_core_mixed_redundant`	M2	$d_W = 4$, $J = 1$, $r_{\text{lin}}^{\text{core}} = 0$	0.306
`ki_3d_mixed_noncomm_core_gap`	M2	$d_W = 3$, $J = 2$, $r_{\text{lin}}^{\text{core}} = 1/15$	0.318
`ki_6d_two_sector_mixed_core_gap`	M2	$d_W = 6$, $J = 2$, $r_{\text{lin}}^{\text{core}} = 3/40$	0.576
`pure_atom_chord_endpoints_U2`	M3	$d_W = 2$, $\|U\| = 2 \le 9$	0.312
`pure_atom_qutrit_erasure_U2`	M3	$d_W = 3$, $\|U\| = 2 \le 19$	0.406
`pure_atom_4d_redundant_U2`	M3	$d_W = 4$, $\|U\| = 2 \le 33$	0.655
`pure_atom_3signal_qubit_U3`	M3	$d_W = 2$, $\|U\| = 3 \le 13$	0.340
`sector_split_erasure_blind_sep`	M4	vis = 0, blind = 1, additive	0.075
`sector_split_two_sector_mixed`	M4	vis = 3/40, blind = 1/2, additive	0.078
`sector_split_three_sector_nonadd`	M4	vis = 9/50, blind = 1/2, non-add	0.082
`sector_split_four_sector_pure_cores`	M4	vis = 0, blind = 3/2, additive	0.091

where global ω_x is mixed yet $r_{\text{lin}}^{\text{core}} = 0$). In `tests/test_suite.py`, independent exact Gaussian-rational $\mathbb{Q}[i]$ 2×2 Hermitian arithmetic cross-checks exact purity $\text{Tr}(M_0^2) = 9/10$ and Frobenius commutator norm $\text{Tr}([M_0, M_1]^\dagger[M_0, M_1]) = \frac{1}{2}\|\vec{r}_0 \times \vec{r}_1\|_2^2$, confirms exact LDL^T PSD rejection of indefinite zero-diagonal matrices and exact commutant-dimension detection (dim = 2) on reducible 3×3 ensembles, and verifies that 57/57 live adversarial forgeries—including Bloch vectors outside the unit ball, non-PSD matrices, reducible 3×3 direct sums, hidden 4×4 redundant factors, partially commuting multi-sector cores, exceeded Carathéodory bounds, and violated Proposition 9 pure-core additivity—raise `KoashiImotoCertificateError` (`3 passed in 0.05s`).

6 Scope, Limitations, and Conclusion

This work machine-checks the finite-dimensional algebraic structure underlying Polloreno's Koashi–Imoto criterion for Holevo-rate visible compression [5], certifying qubit Bloch dichotomies, exact block-tensor KI decompositions, pure-atom Carathéodory mixtures, and sector-splitting/additivity relations over $\mathbb{Q}$. It does *not* formalize the von Neumann entropy continuity bounds, Fawzi–Renner recovery map, or asymptotic decoupling converse in Lean 4, and Mode 2/Mode 3 certificates restrict to real-symmetric rational density matrices (while Mode 1 covers full 3D rational Bloch vectors). Within this exact algebraic scope, our Lean 4 kernel and zero-float `KOASHI-IMOTO-CERT-1.0` auditor deliver sub-millisecond, manifest-locked verification for visible quantum compression witnesses.

References

[1] Leonardo de Moura and Sebastian Ullrich. The Lean 4 theorem prover and programming language. In *Automated Deduction – CADE 28*, volume 12699 of *LNCS*, pages 625–635. Springer, 2021.

[2] Omar Fawzi and Renato Renner. Quantum conditional mutual information and approximate Markov chains. *Communications in Mathematical Physics*, 340(2):575–611, 2015.

[3] Patrick Hayden, Richard Jozsa, Dénes Petz, and Andreas Winter. Structure of states which satisfy strong subadditivity of quantum entropy with equality. *Communications in Mathematical Physics*, 246(2):359–374, 2004.

[4] Masato Koashi and Nobuyuki Imoto. Operations that do not disturb partially known quantum states. *Physical Review A*, 66:022318, 2002.

[5] Anthony M. Polloreno. A Koashi–Imoto criterion for Holevo-rate visible compression. arXiv preprint arXiv:2611.33129, November 2026.

[6] Benjamin Schumacher. Quantum coding. *Physical Review A*, 51(4):2738–2747, 1995.