

A sharp two-rate reset witness against static qutrit disorder

Lluis Eriksson

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Abstract

Can stationary states at a few reset rates distinguish fluctuating dephasing from an arbitrary distribution of static Hamiltonians? For a qubit, every finite list of stationary states of a real dephasing semigroup has a finite static alias. We exhibit a sharp obstruction for a qutrit. If $c_{jk}(\nu)$ is the real, normalized stationary coherence under Poisson resets and $A(\nu) = [c_{12}(\nu) + c_{23}(\nu)]/2$, every static diagonal Hamiltonian ensemble, without support or moment restrictions, satisfies

$$A(a) + 9A(4a) - 5c_{13}(4a) \leq 5 \quad (a > 0).$$

The completely positive semigroup with generator $-\gamma[D, [D, \cdot]]$, $D = \text{diag}(0, 1, 2)$, gives $16/3$ at $a = 2\gamma$. We prove the inequality by an explicit polynomial certificate, determine all its finite equality points, and show that the exact infimum of the uniform error in the three selected statistics is $1/45$. The infimum is approached by seven-point static ensembles but is attained by no probability measure on $\mathbb{R}^2$. A single reset rate cannot exclude static disorder for this semigroup. We also give a deterministic error rule, a finite-shot implementation, and a spectral coupling that extends the exclusion to arbitrary static qutrit Hamiltonian ensembles with noisy population calibration. If all three population leakages at rate a are at most δ , the bound becomes $W_a \leq 5 + 30\sqrt{2}\delta + 30\delta$. This is a witness against static disorder, not against classical time-dependent noise or a proof of a quantum environment.

1 Question and relation to earlier work

Static Hamiltonian ensembles are an established description of inhomogeneous broadening. In particular, Cauchy-distributed qubit frequencies reproduce exponential dephasing exactly [1, 2]. Higher-dimensional dephasing imposes additional consistency: three transition frequencies of a qutrit must be differences of the same three energies. Chen et al. explicitly formulate this joint-distribution constraint using the root system of $\mathfrak{su}(3)$ [3]. The distinction between Hamiltonian ensemble representability and other notions of classicality or Markovianity is also established [4]. Pulse-based discrimination of static broadening is much older: the Hahn echo refocuses static frequency disorder already for a qubit [13]. Our restrictions are free evolution between preparations and stationary or exponentially sampled observations, without echo or decoupling pulses. We claim no advantage over those controlled protocols. Classical stochastic dilations of dephasing semigroups are studied systematically in [14, 8]. Finite-data certification of different environmental properties, including tests based on failure of mixed-unitarity, is already available [10]; our selected statistics do not certify that stronger property.

The question here is narrower and operational: can *two stationary reset settings*, without reconstructing a full time-dependent channel, exclude every static diagonal ensemble? We give an explicit yes answer, an exact bounded-observable inequality, and a sharp distance to the allowed data. The conceptual incompatibility of qutrit marginals is prior work. The candidate contribution is the finite-resolvent witness and its sharp error geometry. The separating-hyperplane method is classical generalized-moment geometry [15], not a new general method of moment

inference. The literature comparison does not establish that nobody has previously derived the same inequality.

Stationary reset states are Laplace transforms of free evolution; this renewal description is standard [5]. Their diagnostic value depends on which controls and preparations are held fixed. The same probability law of static Hamiltonians must be used at all settings. Allowing a different law at each reset rate destroys the proposed test. The author's earlier discussions of maintenance and restoration [11] and finite-data moment falsification [12] motivated this question. These unreviewed manuscripts are contextual only; no theorem from them is used here, and no thermodynamic work claim is made.

2 Protocol and the model being excluded

Set $\hbar = 1$. In each realization draw (X, Y) from an arbitrary Borel probability measure μ on $\mathbb{R}^2$ and hold

$$H_{X,Y} = \text{diag}(0, X, X + Y) \quad (1)$$

fixed. No independence, symmetry, density, finite-support, bandwidth or moment assumption is imposed on μ . Reset the system, independently of its sampled energies, to $\rho_+ = |+_3\rangle\langle+_3|$, where $|+_3\rangle = (|1\rangle + |2\rangle + |3\rangle)/\sqrt{3}$, at Poisson rate $\nu > 0$. The static register is not reset. After taking the stationary limit, average across independently prepared realizations. This is ensemble tomography, not an ergodic time average over a single conserved register.

For a signed energy gap Z , the normalized stationary coherence is

$$z_Z(\nu) = \int_0^\infty \nu e^{-\nu t} \mathbb{E} e^{iZt} dt = \mathbb{E} \frac{\nu}{\nu - iZ}, \quad c_Z(\nu) := \Re z_Z(\nu) = \mathbb{E} g_\nu(Z), \quad g_\nu(z) = \frac{\nu^2}{\nu^2 + z^2}. \quad (2)$$

Boundedness justifies the interchange of expectations and integrals. Operationally this is also a Laplace-weighted free-induction-decay (FID/Ramsey) measurement: prepare independently, draw a waiting time from $\text{Exp}(\nu)$, and measure once. Physical repeated resetting is therefore not necessary to obtain the same averaged data. The preparation and Hamiltonian law must still be the same at every setting; compare the FID formulation in [2]. Thus $c_{jk}(\nu) = 3\Re(\rho_{\mu,\nu})_{jk}$ and

$$c_{12}(\nu) = \mathbb{E} g_\nu(X), \quad c_{23}(\nu) = \mathbb{E} g_\nu(Y), \quad c_{13}(\nu) = \mathbb{E} g_\nu(X + Y), \quad A(\nu) = \frac{c_{12}(\nu) + c_{23}(\nu)}{2}. \quad (3)$$

All populations equal 1/3. Imaginary coherences are allowed in the excluded model; the witness uses their real parts only.

The reference process is the legitimate Lindblad semigroup

$$\mathcal{L}_\gamma(\rho) = -\gamma[D, [D, \rho]], \quad D = \text{diag}(0, 1, 2), \quad \gamma > 0. \quad (4)$$

It has jump operator $\sqrt{2\gamma}D$. Its adjacent coherences decay as $e^{-\gamma t}$ and the outer coherence as $e^{-4\gamma t}$. Under the same reset protocol,

$$A_M(\nu) = \frac{\nu}{\nu + \gamma}, \quad c_{13,M}(\nu) = \frac{\nu}{\nu + 4\gamma}. \quad (5)$$

This semigroup also has a classical *time-dependent* Gaussian noise realization: the random phase $\sqrt{2\gamma}W_t D$, with standard Brownian motion W_t , produces precisely these factors. It follows that rejecting static ensembles does not certify quantum noise, nonclassical dynamics in every usage of that term, or non-Markovianity.

3 The sharp universal inequality

Theorem 1 (Two-rate static-disorder bound). *For every probability measure μ in (1) and every $a > 0$,*

$$W_a := A(a) + 9A(4a) - 5c_{13}(4a) \leq 5. \quad (6)$$

The constant 5 is attained. Equality holds exactly when μ is supported on

$$(0, 0), \quad (a\sqrt{2}, a\sqrt{2}), \quad (-a\sqrt{2}, -a\sqrt{2}). \quad (7)$$

Proof. Rescale $x = X/a$, $y = Y/a$ and write $A_r(x, y) = [g_r(x) + g_r(y)]/2$. It is enough to show

$$h(x, y) := 5 - A_1(x, y) - 9A_4(x, y) + 5g_4(x + y) \geq 0. \quad (8)$$

Let

$$\Delta = (1 + x^2)(16 + x^2)(1 + y^2)(16 + y^2)(16 + (x + y)^2) > 0.$$

Clearing denominators gives the exact identity

$$h(x, y) = \frac{5P(x, y)}{2\Delta}. \quad (9)$$

For $p = xy$ and $d = (x - y)^2$, the polynomial has the form

$$\begin{aligned} P &= (5p^2 + 48)d^2 + 2B(p)d + 8p(p - 2)^2(p + 1)(p + 16), \\ B(p) &= p^4 + 15p^3 + 108\left(p - \frac{46}{27}\right)^2 + \frac{1904}{27}. \end{aligned} \quad (10)$$

If $p \geq 0$, all terms are nonnegative and $B(p) > 0$. Equality requires $d = 0$ and $p \in \{0, 2\}$, giving the three points in (7) after rescaling.

If $p < 0$, set $q = -p > 0$ and $s = (x + y)^2 \geq 0$. The same polynomial is

$$\begin{aligned} P &= (5q^2 + 48)s^2 + (2q^4 + 10q^3 + 216q^2 + 1120q + 768)s \\ &\quad + 64q^4 + 1248q^3 + 3744q^2 + 2560q > 0. \end{aligned} \quad (11)$$

This proves the pointwise inequality and all its zeros. Integrate (8) against μ . Since h is bounded, nonnegative, and vanishes only at the stated points, its integral is zero if and only if the measure is supported there. No moment condition has entered the argument. $\square$

The equality set above consists of finite points. There is also asymptotic saturation along the coordinate axes:

$$h(x, 0) = h(0, x) = \frac{15x^2}{2(1 + x^2)(16 + x^2)} \longrightarrow 0.$$

These escaping directions explain the limiting construction in Theorem 3; they are not additional points in $\mathbb{R}^2$.

Corollary 2 (Violation by a completely positive semigroup). *For (4), put $t = \gamma/a$. Then*

$$W_{a,M} - 5 = \frac{t(7 - 5t)}{(1 + t)(4 + t)}. \quad (12)$$

It is positive for $0 < t < 7/5$, and its maximum over $a > 0$ is $1/3$, attained at $a = 2\gamma$. In particular,

$$A_M(2\gamma) = \frac{2}{3}, \quad A_M(8\gamma) = \frac{8}{9}, \quad c_{13,M}(8\gamma) = \frac{2}{3}, \quad W_{2\gamma,M} = \frac{16}{3}.$$

Proof. Substitute (5). The derivative of the right side of (12) is $-4(2t - 1)(4t + 7)/[(t + 1)^2(t + 4)^2]$. $\square$

The optimizing statement concerns this particular witness family with rate ratio four, not all possible witnesses or reset protocols.

4 An exact error floor and escape to infinity

Use the three selected real statistics, in scaled coordinates,

$$T(\mu) = (\mathbb{E}A_1(X, Y), \mathbb{E}A_4(X, Y), \mathbb{E}g_4(X + Y)), \quad b = \left(\frac{2}{3}, \frac{8}{9}, \frac{2}{3}\right). \quad (13)$$

Here X, Y are dimensionless; physical gaps are aX, aY with $a = 2\gamma$. The vector in (13) deliberately does *not* include $c_{13}(a)$ or all entries of the two stationary states.

Theorem 3 (Sharp distance for the selected statistics). *Over all Borel probability measures on $\mathbb{R}^2$,*

$$\inf_{\mu} \|T(\mu) - b\|_{\infty} = \frac{1}{45}. \quad (14)$$

No measure attains the infimum. It is approached by symmetric seven-point ensembles with unbounded support radius.

Proof. The linear functional $\ell(u) = u_1 + 9u_2 - 5u_3$ has dual norm $\|\ell\|_1 = 15$, and $\ell(b) = 16/3$. Theorem 1 therefore gives $15\|T(\mu) - b\|_{\infty} \geq 1/3$.

For the matching upper bound, assign mass $19/45$ to $(0, 0)$, mass $1/5$ to each of $(\sqrt{2}, \sqrt{2})$ and $(-\sqrt{2}, -\sqrt{2})$, and mass $2/45$ to each of $(0, R), (R, 0), (0, -R), (-R, 0)$, where $R > 0$. These seven masses sum to one. Their statistics are

$$T(\mu_R) = \left(\frac{29}{45} + \frac{4}{45}g_1(R), \quad \frac{13}{15} + \frac{4}{45}g_4(R), \quad \frac{31}{45} + \frac{8}{45}g_4(R)\right). \quad (15)$$

Consequently

$$\|T(\mu_R) - b\|_{\infty} = \frac{1}{45} + \frac{128}{45(16 + R^2)} \rightarrow \frac{1}{45}. \quad (16)$$

For the equality obstruction, attaining $1/45$ forces equality in both the linear bound and the static witness. All three nonzero coefficients then force $T = (29/45, 13/15, 31/45)$. But equality in the static witness requires support on (7); every such measure has its second coordinate at least $8/9$, whereas $13/15 < 8/9$. This is impossible. $\square$

Seven points are an explicit symmetric construction, not an optimal atom count. For the three selected real statistics alone, every attainable vector has a representation with at most four atoms: apply the measurable-function version of Tchakaloff's theorem [16, Corollary 2] to $1, A_1, A_4, g_4(X + Y)$. Such a reduction need not preserve the other or imaginary coherences.

Thus the set of attainable selected data is not closed. This point matters in numerical optimization: an apparent optimum can represent probability mass moving to arbitrarily large energy gaps. On any fixed compact gap domain the probability measures form a compact set and T is continuous, so the best error is attained and is strictly larger than $1/45$. The theorem gives the infimum for the three selected statistics, not a sharp trace-distance error for the full states.

5 Why the second rate and the third level matter

Proposition 4 (One rate is insufficient for the reference example). *For every single rate $\nu > 0$, a two-point static qutrit ensemble reproduces the entire stationary state of (4) exactly. Indeed, take $H = \pm\sqrt{\gamma\nu}D$ with equal probability.*

Proof. For a gap $k\sqrt{\gamma\nu}$ with $k = 1, 2$, the averaged complex coherence is real and equals $g_{\nu}(k\sqrt{\gamma\nu}) = \nu/(\nu + k^2\gamma)$. These are exactly the adjacent and outer coherences; populations agree as well. $\square$

Proposition 5 (Finite qubit aliases). *For a qubit with real Markovian dephasing rate $\gamma > 0$ and any N distinct positive reset rates, a symmetric finite static frequency distribution reproduces all stationary states exactly. One can use $N + 1$ signed frequencies, and fewer are impossible.*

Proof. Let $\nu_1, \dots, \nu_N$ be the rates and write $Q(s) = \prod_j (s + \nu_j) = E(s) + O(s)$, separating even and odd powers. Set

$$F(s) = \begin{cases} O(s)/(sO(s) + \gamma E(s)), & N \text{ odd}, \\ E(s)/(sE(s) + \gamma O(s)), & N \text{ even}. \end{cases} \quad (17)$$

Since $Q(-\nu_j) = 0$, $E(\nu_j) = O(\nu_j)$, giving $F(\nu_j) = 1/(\nu_j + \gamma)$.

The denominator is the degree- $N + 1$ parity part of $(s + \gamma)Q(s)$. For $\omega \geq 0$ the phase

$$\theta(\omega) = \arctan(\omega/\gamma) + \sum_j \arctan(\omega/\nu_j)$$

is strictly increasing from 0 to $(N + 1)\pi/2$. Its crossings of $(k + 1/2)\pi$ when N is odd, or $k\pi$ when N is even, give all the nonnegative imaginary-axis poles. The poles are simple, come in pairs, and include zero exactly when N is even. Their total number is $N + 1$. Each nonzero signed pole has residue

$$p_\omega = \frac{\gamma}{(\gamma^2 + \omega^2)[\gamma/(\gamma^2 + \omega^2) + \sum_j \nu_j/(\nu_j^2 + \omega^2)]} > 0. \quad (18)$$

When present, the zero residue is $p_0 = (1 + \gamma \sum_j \nu_j^{-1})^{-1} > 0$. Also $sF(s) \rightarrow 1$, so the residues sum to one. Partial fractions therefore express F as the Cauchy transform of a symmetric probability measure on these frequencies.

For the lower bound let $v_j(\omega) = (\nu_j + i\omega)^{-1}$ and $f_j = (\nu_j + \gamma)^{-1}$. Any representing measure has the same Gram matrix of $1, v_1, \dots, v_N$, whose lower-right block is $K_{ij} = (f_i + f_j)/(\nu_i + \nu_j)$. Its Schur complement is

$$K - ff^T = 2\gamma \operatorname{diag}(f)C \operatorname{diag}(f), \quad C_{ij} = \frac{1}{\nu_i + \nu_j} = \int_0^\infty e^{-\nu_i t} e^{-\nu_j t} dt > 0.$$

The distinct exponentials are linearly independent, so the Gram matrix has rank $N + 1$. A measure with fewer than $N + 1$ atoms cannot have that rank. $\square$

The construction in Proposition 5 is a specialization of classical positive rational quadrature. In particular Lubinsky's reciprocal-polynomial reproducing kernel [6, equations (1.11)–(1.12)], for weight proportional to $[(\gamma^2 + \omega^2) \prod_j (\nu_j^2 + \omega^2)]^{-1}$, produces the same phase nodes and, after multiplying by $\prod_j (\nu_j^2 + \omega^2)$, the masses (18). We do not claim this quadrature formula as a new discovery; the proof is included to make the qubit/qutrit contrast self-contained. Positive Cauchy transforms and their Gram-matrix constraints also belong to classical Nevanlinna–Pick interpolation; see [17, Theorem 1.2 and the Herglotz representation].

The minimality statement is consequently precise: for real dephasing-semigroup reset data with a fixed dephasing basis, no qubit provides this finite-rate rejection, while the displayed qutrit does; and this particular qutrit cannot be rejected at one rate. It is not a minimality claim over arbitrary controls, echoes, preparations or general non-dephasing processes.

6 Compatibility of the full qutrit semigroup

The finite witness can be related to a simpler full-trajectory fact. The latter is a Cauchy-marginal compatibility problem, within the classical joint-mixability framework [7]; it is not presented as the main novelty of this paper. In particular, the sufficiency follows from Wang

and Wang’s Corollary 3.6 for unimodal symmetric location-scale families. We include an elementary construction and a fractional-moment necessity argument, without applying a finite-mean assumption to Cauchy laws. Mixability without finite means, including Cauchy laws, has also been studied directly in [18]; absence of a mean is not in itself a novel difficulty introduced by this example.

Proposition 6 (Three Cauchy marginals). *For nonnegative real dephasing rates $\Gamma_{12}, \Gamma_{23}, \Gamma_{13}$, a static diagonal ensemble reproduces all factors $e^{-\Gamma_{jk}t}$ for every $t \geq 0$ if and only if the three rates satisfy all triangle inequalities.*

Proof. The characteristic functions, extended to negative times by conjugacy, force $X, Y, X + Y$ to have centered Cauchy laws of scales $\Gamma_{12}, \Gamma_{23}, \Gamma_{13}$, with scale zero interpreted as a point mass. For $0 < p < 1$,

$$|X + Y|^p \leq |X|^p + |Y|^p, \quad \mathbb{E}|\Gamma Z|^p = \Gamma^p \mathbb{E}|Z|^p < \infty$$

for a standard Cauchy Z . Taking expectations and then $p \uparrow 1$ gives $\Gamma_{13} \leq \Gamma_{12} + \Gamma_{23}$. The other two inequalities follow by writing $X = (X + Y) - Y$ and $Y = (X + Y) - X$.

Conversely, let E_1, E_2, E_3 be independent centered Cauchy variables with scales $b_j = (\Gamma_{jk} + \Gamma_{j\ell} - \Gamma_{k\ell})/2 \geq 0$. Then $E_k - E_j$ is centered Cauchy of scale $b_j + b_k = \Gamma_{jk}$. The ensemble $\text{diag}(E_1, E_2, E_3)$, after removing the irrelevant common energy, is of the form (1) and gives all the desired factors. $\square$

The reference rates $(\gamma, \gamma, 4\gamma)$ violate this condition despite generating a completely positive semigroup. For comparison, complete positivity of real qutrit dephasing requires triangle inequalities for the square roots of the rates [9], a different condition. Theorem 1 replaces full-trajectory information by two reset settings and bounded observables for this example.

7 Error control and measurement

Suppose estimates of the selected statistics obey simultaneous deterministic error bounds

$$|\hat{A}(a) - A(a)| \leq \varepsilon_1, \quad |\hat{A}(4a) - A(4a)| \leq \varepsilon_2, \quad |\hat{c}_{13}(4a) - c_{13}(4a)| \leq \varepsilon_3.$$

Every static diagonal model must then satisfy

$$\widehat{W}_a \leq 5 + \varepsilon_1 + 9\varepsilon_2 + 5\varepsilon_3. \quad (19)$$

This is a model-exclusion rule for a given data set. In particular, the ideal reference vector cannot be matched to within uniform error $\varepsilon < 1/45$. If the *estimates themselves* may deviate from the ideal reference by ε , and the same size error bar is used in (19), a conservative worst-case guarantee of rejection requires $\varepsilon < 1/90$. These two different error statements must not be conflated.

For a direct finite-shot implementation, measure

$$M_{jk} = \frac{3}{2}(|j\rangle\langle k| + |k\rangle\langle j|).$$

Its expectation equals c_{jk} and its outcomes lie in $\{-3/2, 0, 3/2\}$. Use independent stationary preparations for the five settings $(12, a), (23, a), (12, 4a), (23, 4a), (13, 4a)$, with m shots per setting. A union bound and Hoeffding’s inequality give simultaneous errors at most e with probability at least $1 - \alpha_c$ whenever

$$m \geq \frac{9}{2e^2} \log \frac{10}{\alpha_c}. \quad (20)$$

The averages defining A then also have error at most e . This conservative bound is not an optimal experimental allocation. Independence here means independently prepared static registers, not repeated correlated measurements on a single conserved register.

8 Robust population calibration for general static ensembles

The sharp witness and distance concern diagonal Hamiltonians. We next give a quantitative reduction from an arbitrary static ensemble to one diagonal ensemble that works simultaneously at every reset rate. For any law μ on Hermitian matrices, define its reset channel by

$$\mathcal{R}_\nu(\rho) = \mathbb{E}_\mu \int_0^\infty \nu e^{-\nu t} e^{-iHt} \rho e^{iHt} dt.$$

Theorem 7 (A common diagonal surrogate). *Let μ be any probability law on finite Hermitian $d \times d$ matrices, and let $\mathcal{R}_\nu$ be its stationary reset channel. Define*

$$L_j(\nu) = 1 - \langle j | \mathcal{R}_\nu(P_j) | j \rangle, \quad P_j = |j\rangle\langle j|.$$

There is one probability law $\tilde{\mu}$ on diagonal Hamiltonians, independent of ν , such that the normalized coherences from the uniform reset state satisfy, for all $j \neq k$ and all $\nu > 0$,

$$|z_{jk,\mu}(\nu) - z_{jk,\tilde{\mu}}(\nu)| \leq \sqrt{d-1}(\sqrt{L_j(\nu)} + \sqrt{L_k(\nu)}) + (d-1)\sqrt{L_j(\nu)L_k(\nu)}. \quad (21)$$

Here $z_{jk,\mu}(\nu) = d\langle j | \mathcal{R}_\nu(\rho_+) | k \rangle$ and ρ_+ is the uniform pure state. No norm or moment bound on the Hamiltonian distribution is needed.

Proof. Conditional on H , let $\eta_j^H(B) = \langle j | P_H(B) | j \rangle$ be its local spectral probability measure. Draw energies $E_1, \dots, E_d$ independently conditional on H , with respective laws η_j^H , and set $\tilde{H} = \text{diag}(E_1, \dots, E_d)$. The resulting diagonal Hamiltonian law is independent of time and reset rate. This construction uses spectral measures and does not require an arbitrary eigenbasis choice at degenerate eigenvalues. For $U = e^{-iHt}$, its conditional free coherence is $U_{jj}\overline{U_{kk}}$.

For the original uniform pure input, the normalized free coherence is $(U_{jj} + s_j)(\overline{U_{kk}} + \overline{s_k})$, where $s_j = \sum_{\ell \neq j} U_{j\ell}$. Put $l_j = 1 - |U_{jj}|^2 \in [0, 1]$. Unitarity and Cauchy-Schwarz give $|s_j| \leq \sqrt{d-1}\sqrt{l_j}$. The same l_j equals the total probability of leaving $|j\rangle$, since the squared norm of both row j and column j is one. Since $|U_{jj}| \leq 1$, the absolute difference of the two coherences is bounded by

$$\sqrt{d-1}(\sqrt{l_j} + \sqrt{l_k}) + (d-1)\sqrt{l_j l_k}.$$

Average over H and over the exponential age distribution $\nu e^{-\nu t} dt$. This is a probability space, and $\mathbb{E} l_j = L_j(\nu)$. Jensen's and Cauchy-Schwarz inequalities give (21). All integrands are bounded, so no moment or dominated-differentiation assumption is necessary. $\square$

Corollary 8 (Noisy population-calibrated qutrit witness). *For any static qutrit Hamiltonian ensemble, if $L_j(a) \leq \delta$ for all $j = 1, 2, 3$, then its uniform-reset data obey*

$$W_a \leq 5 + 30\sqrt{2\delta} + 30\delta. \quad (22)$$

Consequently, with simultaneous coherence error bounds as in (19) and simultaneous certified population bounds $L_j(a) \leq \delta$, observing

$$\widehat{W}_a > 5 + 30\sqrt{2\delta} + 30\delta + \varepsilon_1 + 9\varepsilon_2 + 5\varepsilon_3$$

excludes every static qutrit Hamiltonian ensemble using the same Hamiltonian law for all the tested reset rates and preparations.

Proof. For a fixed Hamiltonian, the population leakage has the positive spectral representation

$$L_j(\nu) = \mathbb{E}_H \iint \frac{(E - E')^2}{\nu^2 + (E - E')^2} d\eta_j^H(E) d\eta_j^H(E'). \quad (23)$$

Indeed, the return amplitude is $U_{jj}(t) = \int e^{-iEt} d\eta_j^H(E)$; expanding its squared modulus and taking the exponential age average gives (23). Thus $L_j(\nu)$ is nonincreasing in ν , without any moment restriction. In particular $L_j(4a) \leq L_j(a) \leq \delta$. Theorem 7 bounds each coherence error relative to its one common diagonal surrogate by $2\sqrt{2\delta} + 2\delta$ at both rates. The same bound holds for the adjacent averages. Apply Theorem 1 to the surrogate and sum the errors:

$$(1 + 9 + 5)(2\sqrt{2\delta} + 2\delta) = 30\sqrt{2\delta} + 30\delta.$$

□

The constants in (22) are conservative, not claimed optimal. The ideal reference has $L_j = 0$. In the absence of coherence error, its margin $1/3$ exceeds the correction whenever

$$\delta < \left(\frac{2\sqrt{115} - 15\sqrt{2}}{30} \right)^2.$$

For finite data, population upper bounds must have their own stated joint confidence. Sampling a fresh register for each shot remains essential; an independent-shot analysis cannot be applied to one uncontrolled conserved register. The sharp $1/45$ distance from Theorem 3 remains a statement about the diagonal model and selected statistics, not a sharp error for this enlarged test.

The practical requirements of this conservative bound are demanding. With no coherence error, the reference is certified only when $\delta < \delta_* \simeq 6.1052 \times 10^{-5}$. With five simultaneous coherence errors at most e , the worst-case reference guarantee is

$$30e + 30\sqrt{2\delta} + 30\delta < \frac{1}{3}.$$

For example, $e = 1/180$ and $\alpha_c = 0.025$ require about 8.74×10^5 shots *per coherence setting* under (20). For each of the three population preparations, n independent shots with zero escapes give the one-sided upper bound $L_j \leq \log(3/\alpha_p)/n$, simultaneously except with probability at most α_p : for any larger leakage, the probability of zero escapes is at most e^{-nL_j} , and a union bound covers the three states. Taking $\alpha_p = 0.025$ requires about 4.79×10^5 zero-escape shots per state to certify $\delta \leq 10^{-5}$. The combined coverage is at least $1 - \alpha_c - \alpha_p$. If escapes occur, a binomial confidence bound must replace the zero-event formula. These estimates account for sampling only, not state-preparation or measurement bias, and are not evidence that an efficient experiment has been achieved.

Exact data admit a simpler calibration with only two basis states.

Proposition 9 (Exact population calibration). *Let μ be a probability measure on arbitrary finite Hermitian 3×3 matrices, with $\mathcal{R}_\nu$ as defined above. If, at any one $\nu > 0$, $\mathcal{R}_\nu(P_1) = P_1$ and $\mathcal{R}_\nu(P_2) = P_2$, where $P_j = |j\rangle\langle j|$, then H is diagonal almost surely. Consequently, these two exact calibrations together with $W_a > 5$ exclude every static qutrit Hamiltonian ensemble with the same Hamiltonian law for all reset preparations and rates.*

Proof. For each $j = 1, 2$ and $k \neq j$, the assumption gives

$$0 = \mathbb{E}_\mu \int_0^\infty \nu e^{-\nu t} |\langle k | e^{-iHt} | j \rangle|^2 dt.$$

The integrand is nonnegative. For almost every H , all these finitely many amplitudes vanish for almost every t , and hence for every t by continuity. Differentiating at zero gives $H_{kj} = 0$. Hermiticity and dimension three leave no off-diagonal entry. □

The reference semigroup passes these population calibrations exactly. Neither the exact nor the noisy calibration excludes arbitrary system–bath Hamiltonians with a quantum bath or preparation-dependent changes in the Hamiltonian distribution.

9 Verification, scope and open questions

The accompanying verifier checks the rational identity (9), both polynomial representations, the equality points, the semigroup violation and its optimizing rate, and the exact seven-point construction using symbolic arithmetic. A separate numerical stress test checks the common diagonal surrogate for 1000 Hermitian examples of dimensions two through six. This numerical check supplements the analytic proof and is not a substitute for it. Numerical searches helped locate the witness; the theorems above do not rely on a finite grid, floating-point positivity, a fitted polynomial, or a semidefinite solver's status.

The concrete advance proposed here is a finite-rate and quantitatively sharp compatibility test. Earlier Hamiltonian-ensemble work supplies the conceptual question, and classical rational quadrature explains the contrasting qubit aliases. The paper does not solve general quantum process identification, establish a microscopic source of noise, or assert a universal boundary for all dephasing generators. There is no experimental validation in this manuscript. Noise with a finite correlation time is a separate interpolation problem: rejection of a static law alone does not estimate that correlation time or identify an Ornstein–Uhlenbeck process. Echo-based methods and non-random-unitary dephasing tests [13, 19] address different control or model classes.

Natural next problems are to optimize over other rate ratios and measurement budgets, to determine the sharp distance for the full two-state data, and to sharpen the population-calibration constants. Higher-dimensional energy-difference constraints may supply further finite resolvent witnesses. These are open directions, not results claimed here.

AI assistance and reproducibility. AI systems assisted the literature search, exploratory computation, proof development and writing. A GPT-6 Astra advisory review contributed literature suggestions and the spectral-monotonicity sharpening of the population bound; it is not a blind independent review. A separate Claude Opus 5.5 review of the preceding draft found no fatal error but rated its significance substantially lower. Both reports and a response are preserved with the source; the present editorial revision has not received a new score. Reviews are advisory and are not formal proof certificates or evidence of scientific priority. The companion source and verification script make every algebraic claim in the main witness auditable. The manuscript is an unsubmitted candidate; no publication decision or external peer-review outcome is implied.

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