

Exact Multiplicity Floors for Dual Singularities from Absorbing Gauss Fibres

Lluis Eriksson

Independent researcher

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Abstract

Let $X^d \subset \mathbb{P}_{\mathbb{C}}^{d+1}$ be a smooth hypersurface, let $\gamma_X : X \rightarrow X^\vee$ be its Gauss map, and let Z be the reduced fibre over a tangent hyperplane W . In the complete $\mathcal{O}_X(m)$ embedding, assume that the span of Z contains the order- s osculating space at every support, where $1 \leq s \leq m$. We prove the sharp dual-multiplicity floor

$$\text{mult}_W(X^\vee) \geq s^d |Z| \geq s^d \binom{d+m}{d}.$$

The first inequality is local and forced by absorption: the equation of W restricts to order at least $s+1$ at every point of the fibre. Each isolated singularity of $X \cap W$ therefore has Milnor number at least s^d , while the classical multiplicity–Milnor formula of Dimca, subsequently extended by Parusiński, identifies the multiplicity of the dual with the sum of these Milnor numbers. The number of analytic branches of $X^\vee$ at W is $|Z|$, so it has the same binomial lower floor. When $m \geq 2s+1$, we also show that the rank-sensitive term from the predecessor paper supplies no stricter inequality here, because a Gauss fibre lies in one hyperplane.

The uniform bounds are attained simultaneously. For every (d, m, s) we construct a smooth integral hypersurface with exactly $\binom{d+m}{d}$ points in one Gauss fibre; their point span is proper and order- s absorbing, and the corresponding hyperplane section has an ordinary isolated singularity of multiplicity $s+1$ at each point and no others. The dual point has exactly that many branches, affine tangent-cone cycle $s^d \sum_{p \in Z} [T_{[W]}(p^\perp)]$, and multiplicity $s^d \binom{d+m}{d}$. For $s=1$ its tangent cone is a reduced arrangement of distinct hyperplanes. The construction is explicit only up to nonempty open choices; no minimal hypersurface degree, analytic classification, or exhaustive priority claim is made.

1 Statement and scope

Let $X^d \subset \mathbb{P}_{\mathbb{C}}^{d+1}$ be a smooth hypersurface of degree $D \geq 2$, let $H = \mathcal{O}_X(1)$, and let

$$\gamma_X : X \longrightarrow (\mathbb{P}^{d+1})^\vee$$

be the Gauss morphism. Its image is the dual hypersurface $X^\vee$. For $\eta \in X^\vee$, write $W_\eta \subset \mathbb{P}^{d+1}$ for the corresponding hyperplane and

$$Z_\eta = (\gamma_X^{-1}(\eta))_{\text{red}}.$$

Fix $m \geq 1$. For a nonempty finite reduced $Z \subset X$, let

$$S_Z = \text{Im} \left(H^0(Z, H^m|_Z)^* \longrightarrow H^0(X, H^m)^* \right).$$

For $p \in X$ and $a \geq 0$, principal-parts evaluation gives the affine osculating block

$$\widehat{\text{Osc}}_p^a(H^m) = \text{Im} \left(H^0((a+1)p, H^m|_{(a+1)p})^* \longrightarrow H^0(X, H^m)^* \right).$$

Definition 1.1. For $1 \leq s \leq m$, the set Z is *point-span s -osculating-absorbing* if

$$\widehat{\text{Osc}}_p^s(H^m) \subseteq S_Z \quad (p \in Z).$$

Put

$$N_{d,m} = \binom{d+m}{d}.$$

Theorem 1.2 (Sharp dual-multiplicity floor). *Let $\eta \in X^\vee$ and $Z = Z_\eta$. If Z is point-span s -osculating-absorbing for the complete H^m embedding, then:*

1. $X^\vee$ has exactly $|Z|$ analytic branches at η and

$$|Z| \geq N_{d,m};$$

2. for every $p \in Z$, the germ of $X \cap W_\eta$ has multiplicity at least $s+1$ and Milnor number $\mu_p \geq s^d$;

- 3.

$$\text{mult}_\eta(X^\vee) = \sum_{p \in Z} \mu_p \geq s^d |Z| \geq s^d N_{d,m};$$

4. as a cycle in $T_\eta((\mathbb{P}^{d+1})^\vee)$, the affine tangent cone is

$$C_\eta(X^\vee) = \sum_{p \in Z} \mu_p [T_\eta(p^\perp)].$$

Theorem 1.3 (Simultaneous sharpness). *For every $d, m \geq 1$, every $1 \leq s \leq m$, and every*

$$D \geq (s+2)N_{d,m} + 1,$$

there exist a smooth integral degree- D hypersurface $X^d \subset \mathbb{P}^{d+1}$, a hyperplane W , and $Z = (\gamma_X^{-1}([W]))_{\text{red}}$ such that

$$|Z| = \dim S_Z = N_{d,m} < h^0(X, H^m),$$

Z is point-span s -osculating-absorbing, and $X \cap W$ has precisely the points of Z as singularities. At every $p \in Z$ the section singularity is ordinary of multiplicity $s+1$, so

$$\mu_p = s^d, \quad \text{mult}_{[W]}(X^\vee) = s^d N_{d,m},$$

and

$$C_{[W]}(X^\vee) = s^d \sum_{p \in Z} [T_{[W]}(p^\perp)].$$

In particular, every inequality in Theorem 1.2 is sharp. When $s = 1$, the $N_{d,m}$ branches are smooth and the tangent cone is the reduced union of the distinct hyperplanes $p^\perp$.

The multiplicity identity used here is classical [1, 4]; Dimca–Ilardi use the same bridge to study nodal hyperplane sections and normal-crossing points of duals [3]. Accordingly, the factor s^d is presented as an explicit local consequence of that classical theory, not as a new general principle about dual multiplicities. The contribution isolated here is the combination with point-span absorption: the resulting sharp binomial floor and the exact dual geometry of equality examples with proper point span. This comparison is selective, not a priority certification.

2 Gauss fibres and branches

Lemma 2.1 (Normalization by the Gauss map). *The Gauss morphism $\gamma_X : X \rightarrow X^\vee$ is finite and birational. Hence it is the normalization of $X^\vee$, and the number of analytic branches of $X^\vee$ at η equals $|Z_\eta|$.*

Proof. If F defines X , its first partial derivatives have no common zero on X because X is smooth. They define γ_X and give

$$\gamma_X^* \mathcal{O}_{X^\vee}(1) = \mathcal{O}_X(D-1),$$

which is ample. A positive-dimensional fibre would make this line bundle trivial on a positive-dimensional complete subvariety, contradicting ampleness. Thus γ_X is finite. Projective biduality identifies the unique tangency point over a general smooth point of $X^\vee$, so the map is birational. Since X is smooth and therefore normal, it is the normalization. Complex algebraic local rings are excellent, normalization commutes with completion, and each completed local ring above η is a power-series domain. The normalization fibre therefore indexes the analytic branches. $\square$

The equality

$$\text{Sing}(X \cap W_\eta) = Z_\eta$$

holds set-theoretically: a point of the hyperplane section is singular exactly when W_η is tangent to X there. Lemma 2.1 makes this set finite, so all section singularities are isolated.

3 Absorption forces high contact

Lemma 3.1 (Local annihilator consequence). *If Z is point-span s -osculating-absorbing and $a \in H^0(X, \mathcal{I}_Z \otimes H^m)$, then*

$$a_p \in \mathfrak{m}_{X,p}^{s+1} H_p^m \quad (p \in Z).$$

Proof. The annihilator of S_Z is $H^0(X, \mathcal{I}_Z \otimes H^m)$. The annihilator of $\widehat{\text{Osc}}_p^s(H^m)$ consists of sections whose germ vanishes modulo $\mathfrak{m}_{X,p}^{s+1}$. The inclusion $\widehat{\text{Osc}}_p^s(H^m) \subseteq S_Z$ reverses under annihilators and gives the claim. $\square$

Proposition 3.2 (Contact along an absorbing Gauss fibre). *Let ℓ_η be a linear equation for W_η . Under the hypotheses of Theorem 1.2,*

$$(\ell_\eta|_X)_p \in \mathfrak{m}_{X,p}^{s+1} H_p \quad (p \in Z).$$

Proof. Every point of Z lies in W_η . Fix $p \in Z$ and choose $g_p \in H^0(X, H^{m-1})$ nonzero at p , taking $g_p = 1$ when $m = 1$. The section $\ell_\eta g_p$ of H^m vanishes at every support of Z . Lemma 3.1 puts its germ in $\mathfrak{m}_{X,p}^{s+1} H_p^m$. The germ of g_p is a unit, so division gives the displayed contact. $\square$

This step explains why the factor s^d in Theorem 1.2 is not an extra geometric hypothesis. It is forced by the combination of a common tangent hyperplane and order- s absorption.

4 Milnor numbers and the dual multiplicity

Lemma 4.1 (Milnor floor from order). *Let $R = \mathbb{C}\{x_1, \dots, x_d\}$ with maximal ideal $\mathfrak{m}$, and let $f \in \mathfrak{m}^{s+1}$ have an isolated critical point at the origin. Then*

$$\mu(f) = \dim_{\mathbb{C}} R / \left(\frac{\partial f}{\partial x_1}, \dots, \frac{\partial f}{\partial x_d} \right) \geq s^d.$$

Equality holds if the initial homogeneous form f_{s+1} defines a smooth hypersurface in $\mathbb{P}^{d-1}$ (with the empty hypersurface convention for $d = 1$).

Proof. Let J be the Jacobian ideal. Isolatedness makes J $\mathfrak{m}$ -primary. It is generated by a system of parameters in the regular local ring R , so $\mu(f) = \text{length}(R/J) = e(J)$. Since $J \subseteq \mathfrak{m}^s$, monotonicity of Hilbert–Samuel multiplicity gives

$$e(J) \geq e(\mathfrak{m}^s) = s^d.$$

If f_{s+1} is smooth projectively, its d partial derivatives are a homogeneous regular sequence of degree s . The associated-graded intersection has length s^d , so the local Jacobian colength is s^d . $\square$

Dimca’s multiplicity formula [1] says that for a smooth complex hypersurface and a hyperplane section with isolated singularities,

$$\text{mult}_\eta(X^\vee) = \sum_{p \in \text{Sing}(X \cap W_\eta)} \mu_p. \quad (1)$$

The refined tangent-cone statement [2, Proposition 11.24] is

$$C_\eta(X^\vee) = \sum_p \mu_p [T_\eta(p^\perp)]. \quad (2)$$

Here $p^\perp$ is the hyperplane through η in the dual projective space corresponding, under biduality, to p ; the displayed component is its linear tangent space at η . Equivalently, projectivizing gives the cycle $\sum_p \mu_p [\mathbb{P}(T_\eta(p^\perp))]$ in the projectivized ambient tangent space.

For completeness, the branch contribution can also be read directly from the Gauss map. Choose analytic coordinates at p in which X is the graph

$$x_{d+1} = h(z_1, \dots, z_d), \quad h \in \mathfrak{m}^{s+1},$$

and $W_\eta = (x_{d+1} = 0)$. Work in the analytic local rings $A = \mathcal{O}_{X^\vee, \eta}^{\text{an}}$ and $R = \mathcal{O}_{X, p}^{\text{an}} \cong \mathbb{C}\{z_1, \dots, z_d\}$, and put $I = \mathfrak{m}_\eta R$. The local expression for the Gauss map gives, up to signs,

$$I = \left(h_1, \dots, h_d, h - \sum_{i=1}^d z_i h_i \right), \quad h_i = \frac{\partial h}{\partial z_i}. \quad (3)$$

Thus, for $J = (h_1, \dots, h_d)$,

$$J \subseteq I \subseteq \mathfrak{m}^s.$$

When the initial form h_{s+1} is projectively smooth, J is a parameter ideal with $e(J) = s^d$. Monotonicity and $e(\mathfrak{m}^s) = s^d$ then give

$$s^d = e(J) \geq e(I) \geq e(\mathfrak{m}^s) = s^d,$$

so the contribution of that normalization branch is exactly s^d . The finite-extension associativity formula for Hilbert–Samuel multiplicity recovers the sum over the points above η ; see [11, Chapter VIII, §10, Corollary 1]. Analytification and completion preserve the multiplicities in question. This is a local recovery of the classical multiplicity–Milnor formula in this ordinary case, included to make the equality mechanism in the examples explicit.

Proof of Theorem 1.2. Lemma 2.1 gives the branch count. Order- s absorption contains tangent absorption, so the exact point-span tangent floor [5, Theorem 1.1] gives

$$|Z| \geq N_{d, m}.$$

Proposition 3.2 says that a local equation of the hyperplane section belongs to $\mathfrak{m}_{X, p}^{s+1}$. The critical point is isolated, hence Lemma 4.1 gives $\mu_p \geq s^d$. Equations (1) and (2) give all remaining assertions. $\square$

Remark 4.2 (Why the earlier rank term gives no refinement here). Suppose $m \geq 2s + 1$. Because $Z \subset W_\eta$, the equation of W_η lies in the kernel of linear evaluation, so

$$r_1(Z) := \text{rk}(H^0(X, H) \rightarrow H^0(Z, H|_Z)) \leq d + 1.$$

The quotient

$$\frac{\binom{d+2s+1}{d}}{\binom{d+s}{d}} = \prod_{j=1}^d \frac{2s+1+j}{s+j}$$

is nondecreasing in $s \geq 1$ term by term. At $s = 1$ it equals $(d+3)(d+2)/6 \geq d+1$. Hence

$$\binom{d+m}{d} \geq (d+1) \binom{d+s}{d} \geq r_1(Z) \binom{d+s}{d}.$$

Thus the rank-sensitive maximum in [6, Theorem 1.3] specializes exactly to $N_{d,m}$ for a Gauss fibre; presenting it as a stronger bound here would be misleading.

5 Interpolation with prescribed ordinary jets

We now adapt the proper-span construction of [6]; its antecedent for ordinary double points is the author's public proof note [8]. The interpolation and extension architecture $F = f + yG$ is therefore not claimed as new here. The added points are the prescribed ordinary initial forms, the exact Gauss-fibre identity, and the resulting branch and dual-multiplicity calculation.

Let $W \cong \mathbb{P}^d$ and choose $N = N_{d,m}$ distinct points $Z \subset W$ such that

$$H^0(W, \mathcal{O}_W(m)) \xrightarrow{\sim} H^0(Z, \mathcal{O}_Z(m)). \quad (4)$$

Such sets exist because the evaluation lines of all points span the dual of $H^0(W, \mathcal{O}_W(m))$; select a basis. Very ampleness separates the supports.

Lemma 5.1 (Simultaneous jets from separators). *Let $p_1, \dots, p_t$ be distinct points of $\mathbb{P}^d$ and let $a_i \geq 0$. Restriction to the disjoint fat points is surjective in every degree*

$$k \geq \sum_{i=1}^t (a_i + 1) - 1 :$$

$$H^0(\mathbb{P}^d, \mathcal{O}(k)) \longrightarrow \bigoplus_{i=1}^t H^0(\mathcal{O}(k) \otimes \mathcal{O}_{\mathbb{P}^d, p_i} / \mathfrak{m}_{p_i}^{a_i+1}).$$

Proof. For $i \neq j$, choose a linear form ℓ_{ij} vanishing at p_j and nonzero at p_i . The product

$$P_i = \prod_{j \neq i} \ell_{ij}^{a_j+1}$$

kills every other target and is a unit at p_i . Degree- a_i forms realize all order- a_i jets at p_i . Their products with P_i have common degree $\sum_j (a_j + 1) - 1$ and isolate arbitrary data at p_i . Summing over i proves surjectivity in that degree. Multiplication by a form nonzero at all supports raises the degree. $\square$

For every $p \in Z$, choose a homogeneous form

$$q_p \in \text{Sym}^{s+1}(\mathfrak{m}_{W,p} / \mathfrak{m}_{W,p}^2)$$

whose projective zero locus in $\mathbb{P}(T_p W)$ is smooth. Put

$$E = (s+2)N,$$

and fix $D \geq E + 1$. Lemma 5.1 supplies $f_0 \in H^0(W, \mathcal{O}_W(D))$ whose class at every p is q_p modulo $\mathfrak{m}_{W,p}^{s+2}$ and whose lower terms vanish. Let

$$V = H^0(W, \mathcal{I}_{(s+2)Z}(D)), \quad \mathcal{A} = f_0 + V.$$

Proposition 5.2 (A section with exactly the prescribed singularities). *There is $f \in \mathcal{A}$ such that $V(f) \subset W$ is smooth away from Z . At every $p \in Z$ it has an ordinary isolated hypersurface singularity of multiplicity $s + 1$ and Milnor number s^d .*

Proof. Fix $q \in W \setminus Z$. For every $p \in Z$, choose a hyperplane $\ell_{p,q}$ through p and not through q , and set

$$A_q = \prod_{p \in Z} \ell_{p,q}^{s+2}.$$

It has degree E , belongs to the ideal of $(s + 2)Z$, and is a unit on the first neighbourhood $2q$. Since $D - E \geq 1$, multiplication by A_q proves that

$$V \longrightarrow H^0(2q, \mathcal{O}_{2q}(D))$$

is surjective. Therefore the affine condition $j_q^1 f = 0$ has codimension $d + 1$ in $\mathcal{A}$. Its incidence over the d -dimensional base $W \setminus Z$ has dimension at most $\dim \mathcal{A} - 1$. Choose f outside the closure of its image. The fixed jets q_p give the asserted ordinary singularities, and Lemma 4.1 gives $\mu_p = s^d$. $\square$

6 Smooth hypersurfaces and exact Gauss fibres

Embed $W = V(y)$ as a hyperplane in $\mathbb{P}^{d+1}$ and lift f independently of y . Put

$$U = H^0(\mathbb{P}^{d+1}, \mathcal{O}(D - 1)), \quad F_G = f + yG \quad (G \in U).$$

Proposition 6.1 (Smooth extension). *There is $G \in U$, nonzero at every point of Z , such that $X = V(F_G)$ is a smooth integral hypersurface. Moreover*

$$\text{Sing}(X \cap W) = Z, \quad (\gamma_X^{-1}([W]))_{\text{red}} = Z.$$

Proof. On $y \neq 0$, multiplication by y is a unit on first neighbourhoods and $\mathcal{O}(D - 1)$ generates first jets. For fixed q in that $(d + 1)$ -dimensional open set, the affine condition $j_q^1(f + yG) = 0$ has codimension $d + 2$ in U . The corresponding incidence has dimension at most $\dim U - 1$, so the closure of its image is proper. Avoid it together with the finitely many hyperplanes $G(p) = 0$.

The resulting X is smooth on $y \neq 0$. At a point of $W \setminus Z$ lying on X , the tangential first jet of f is nonzero by Proposition 5.2. At $p \in Z$, the normal first jet is $G(p) \neq 0$. Thus X is smooth everywhere. The hypersurface sequence and $H^1(\mathbb{P}^{d+1}, \mathcal{O}(-D)) = 0$ give $H^0(X, \mathcal{O}_X) = \mathbb{C}$, so X is connected; a smooth connected scheme is integral.

The hyperplane section is $V(f)$, hence has singular locus exactly Z . A point of $X \cap W$ belongs to the Gauss fibre over $[W]$ precisely when the section is singular there. A point outside W cannot have W as tangent hyperplane because a tangent hyperplane contains its point. This proves the exact fibre identity. $\square$

At $p \in Z$, the local equation on X is

$$y = -f/G \in \mathfrak{m}_{X,p}^{s+1},$$

and its initial form is the nonzero scalar multiple $-q_p/G(p)$. Thus $X \cap W$ has precisely the ordinary singularities already prescribed.

Proof of Theorem 1.3. Use Proposition 6.1. Since $D > m$, restriction is an isomorphism

$$H^0(\mathbb{P}^{d+1}, \mathcal{O}(m)) \xrightarrow{\sim} H^0(X, \mathcal{O}_X(m)).$$

Let a section on X vanish on Z and let Q be its ambient lift. By (4), $Q|_W = 0$, so $Q = yR$. Since $y \in \mathfrak{m}_{X,p}^{s+1}$ at every support, Q vanishes on $(s+1)Z$. The reverse kernel inclusion is automatic, hence the annihilator criterion proves order- s absorption. Unisolvence gives

$$|Z| = \dim S_Z = N.$$

The ambient degree- m space has dimension $\binom{d+m+1}{d+1} > N$, so the point span is proper.

The exact Gauss fibre is Proposition 6.1; the section singularities have Milnor number s^d . Dimca's formulas (1)–(2) give the stated dual multiplicity and tangent-cone cycle. When $s = 1$, each section singularity is A_1 , the Gauss map is immersive at its support, and the corresponding dual branch is smooth. The tangent hyperplanes $p^\perp$ are distinct because the supports are distinct. $\square$

7 Context, reproducibility, and limitations

The identity $\text{mult}_W(X^\vee) = \sum \mu_p$ and its tangent-cone refinement are not new; they are attributed respectively to Dimca [1] and [2, Proposition 11.24], with broader multiplicity results due to Parusiński [4], and belong to the classical projective-duality and discriminant framework [10, 9]. Dimca–Ilardi prove that a generic smooth hypersurface in $\mathbb{P}^n$, $n \geq 3$, has a dual normal-crossing point of multiplicity n , arising from n nodes in a hyperplane section [3]. They also emphasize the classical fact that isolated singularity hyperplane sections can be extended to smooth hypersurfaces. The present construction uses that freedom subject to the additional unisolvence and absorption constraints.

The normalizing role of the Gauss map and the earlier large-fibre construction also occur in the author's public notes [7, 8]. Those notes do not state the branch count or dual multiplicity computed here; they are nevertheless direct authorial antecedents rather than independent validation. The exact tangent floor is invoked from [5]; the higher-block estimate and the underlying proper-span construction are invoked from [6], to which the exact reduced Gauss-fibre and dual-singularity calculations are added here. These are papers by the same author and are not independent external validation.

The accompanying replay `repro/verify_dual_multiplicity.py` checks finite monomial Jacobian quotients, 105 binomial/multiplicity cases, 100 rank-collapse cases, and selected simplex evaluation matrices using integer and rational arithmetic. The byte-stable runner is `repro/run_all_replays.py`. These scripts do not mechanize the incidence arguments, projective duality, Dimca's formula, or literature priority.

The limitations are material.

- The dual-singularity theorem is over $\mathbb{C}$. No positive-characteristic Milnor, inseparable Gauss-map, or wild-ramification extension is asserted.
- The support is the complete reduced Gauss fibre, not an arbitrary subset. Its scheme-theoretic fibre can be nonreduced and is not classified.
- The degree $D \geq (s+2)N_{d,m} + 1$ is a transparent sufficient threshold, not a minimality result.
- For $s = 1$ the tangent cone is a reduced arrangement of distinct hyperplanes. It is not called a simple normal-crossing divisor when the number of branches exceeds the ambient dimension.
- No analytic classification of the dual branches, equality classification, exhaustive priority review, human peer review, or formal verification is claimed.

8 Conclusion

Osculating absorption in a Gauss fibre forces more than many coincident tangent points. It forces the common tangent hyperplane to have order $s + 1$ contact at every support, converting the exact point-span rank floor into the sharp dual-singularity floor

$$\mathrm{mult}_W(X^\vee) \geq s^d \binom{d+m}{d}.$$

The previously developed prescribed-jet construction realizes equality while keeping the point span proper and the reduced Gauss fibre exact; the new use made of it here is the dual-singularity calculation. The result ties finite reduced span rank, osculating contact, Milnor numbers, normalization branches, and dual multiplicity in one sharp construction. Minimal ambient degree and the geometry of equality configurations remain open.

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