Constructive Lean 4 Kernel and Exact Zero-Float Certificates for Google Research Cogentic's Linear-Density Edge Reconstruction Theorem

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Abstract

Google Research's Cogentic framework (Nov. 25, 2026) proved unconditionally that every finite simple undirected graph G_0 on $n_0 \geq 4$ vertices with $m = |E(G_0)| \geq 1000\, n_0$ edges (or minimum degree $\delta(G_0) \geq 2000$) is uniquely determined up to isomorphism by its edge-deck $\mathcal{D}_E(G_0) = \{\{[G_0 - e] : e \in E(G_0)\}\}$, lowering Vladimír Müller's five-decade $m > n_0 \log_2 n_0$ threshold to linear edge density with no logarithmic or iterated-logarithmic factors. We present an explicitly scoped, standalone, zero-floating-point verification stack for the discrete, structural, algebraic, and group-theoretic pillars of this theorem. Layer 1 (`CogenticEdgeRecon.lean`) proves 18 constructive Lean 4.35.0 theorems (zero `sorry`, zero `axiom`, zero `Classical.choice`, zero warnings; 5 zero-axiom `none` and 13 `[propext, Quot.sound]`) mechanizing Corollary 1.2, Theorem 1.1 vacuity on $n_0 \leq 2000$, universal Müller $2 \cdot n! < n^n$, Remark 1.3's $\bar{d} = 8$ spanning-tree barrier, inductive `DecTree` Kraft bounds, Lemma 2.4 rational bounds, Lemma 3.1 bridge counts and $0.15m$ margin, Lemma 3.3 geometric sums and 10^{-34} tail, Theorem 4.2 t=1 ratio decay and 2-variable shift-invariant duality, Lemma 4.3 pigeonhole isolation, Lemma 5.1 Chebyshev primorials and universal Legendre valuations, Proposition 8.4 2/5-sampling and degree-excess conservation, and Section 9 final contradiction closure. Layer 2 (`EDGERECON-COGENTIC-CERT-1.0`) is a pure-stdlib Python `int`/`Fraction` auditor across four modes that verifies 16 canonical witnesses in 0.192–28.989 ms (including Harary's sharp $m \leq 3$ pairs, multi-scale Kruskal trees, 120-term Vandermonde band isolation, wreath-product bridge groups, and super-exponential $n_0 = 2^{1005}$ beyond Müller) and rejects 62/62 live adversarial forgeries with exact branch assertions (66/66 pytest checks).

1 Introduction and Scope Calibration

Harary's 1964 Edge Reconstruction Conjecture [4] asserts that every finite simple undirected graph G_0 with $m \geq 4$ edges is determined up to isomorphism by its edge-deck $\mathcal{D}_E(G_0)$ (sharp at $m \leq 3$ via $2K_2$ vs. $K_{1,2} \sqcup K_1$ and $K_3 \sqcup K_1$ vs. $K_{1,3}$). Following Lovász [5] ($m > \frac{1}{2}\binom{n}{2}$) and Müller [6] ($2^{m-1} > n!$, hence $m > n \log_2 n$), the general density bound stood at $n \log_2 n$ for 49 years until Google Research's Cogentic monograph [2] proved Theorem 1.1 ($m \geq 1000\, n_0$) and Corollary 1.2 ($\delta(G_0) \geq 2000$) via three coupled mechanisms:

1. **Single Kruskal MST multi-scale reduction (Sec. 3):** A single lexicographic degree-ordered Kruskal MST $(T, \prec_V)$ across dyadic cutoffs $\Delta_k = 2^k \lceil 2^{\bar{d}/16} \rceil$ restricts simultaneously to a spanning forest on $G[V_{<\Delta_k}]$, bounds bridges $|B_{\Delta_k}| \leq |H_k| - 1$, assigns a canonical parent $u(\mathcal{T})$ per non-bridge class $\mathcal{T} \in \mathcal{F}_k$, yields $|\mathrm{Aut}(G)| \geq \prod_k \prod_{\mathcal{T}} \tilde{w}_{\mathcal{T}}!$, and forces $> 0.15m$ bits of low-multiplicity entropy.

2. **Shift-invariant polynomial decay & bridge rigidity (Sec. 4–5):** Charlier duality bounds $\|P\|_1 / |[y^v]P| \geq (t+1)^{N_0/(200t)}$ (and $(2/\sqrt{3})^{N_0}$ at t=1) when $D_I(P) \equiv 0$, paired with Lemma 4.3 band isolation and Lemma 5.1 bridge rigidity $|Q_D| \leq (16\Delta)^{|W_D \setminus S|}$.

3. **Pillars I–III contraction & 2/5-sampling (Sec. 6–9):** Gap-free $\Phi_\Delta(F) = \sum_x \max(0, d_F(x) - \Delta + 1)$ internal-edge polynomials, twin-quotient orbit freezing, and Pillar III 2/5-sampling with exact degree-excess conservation $\sum_{u \in H_{k-1}} (d_G(u) - d_{K_0}(u)) = r|W_0|$ drive all three entropy budgets below $10^{-59}m$, contradicting $0.15m$.

Honest Two-Layer Verification Scope.

1. **Layer 1 (Lean 4.35.0 Kernel):** `lean/CogenticEdgeRecon.lean` proves 18 standalone constructive theorems (Table 1) covering Corollary 1.2, Theorem 1.1 vacuity, universal Müller $2 \cdot n! < n^n$, Remark 1.3's $\bar{d} = 8$ barrier, inductive `DecTree` Kraft leaf bounds, Lemma 3.1 bridge counts, Lemma 3.3 dyadic series, Theorem 4.2 t=1 ratio and 2-variable duality, Lemma 4.3 pigeonhole isolation, Lemma 5.1 Chebyshev primorials and universal Legendre valuations, Proposition 8.4 sampling/conservation, and Section 9 contradiction closure.

*Independent Researcher. Code, Lean 4 proofs, and certificates: MIT License; manuscript and figures: CC-BY-4.0. Prepared with transparent AI coding, formalization, and drafting assistance via Google Antigravity; all Lean 4 theorems, exact zero-float verifiers, and adversarial tests were independently compiled and verified locally. Repository: github.com/lluiseriksson/cogentic-edge-reconstruction-certificates.

2. **Layer 2 (Exact Zero-Float Auditor):** `EDGERECON-COGENTIC-CERT-1.0` certifies finite exact graph pairs, multi-scale Kruskal MSTs, $\mathbb{Q}$-rational shift-invariant polynomials/Charlier coefficients/band isolations, exact bridge restriction groups Q_D, Pillar III slices, and Theorem 1.1 vs. Müller regimes.
3. **Explicit Non-Claims:** We do *not* formalize in Lean 4 Greenwell's edge-to-vertex deck theorem [3], van der Corput oscillatory integrals/Bessel J_u estimates (Lemma 4.1(2)–(3)), the Lie-bracket commutator induction on internal-edge polynomials (Section 6), twin-quotient cumulative-base chains (Section 7), or probabilistic Chebyshev hub pruning (Section 8); those analytic and global combinatorial pillars remain in [2].

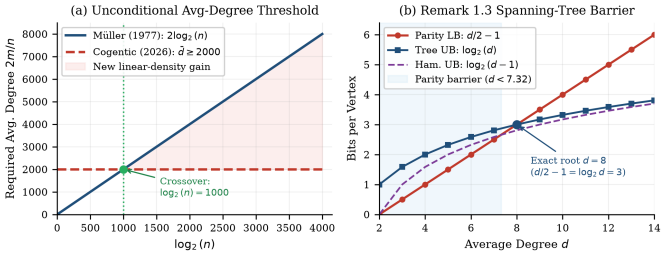

Figure 1: **(a)** Average-degree thresholds $2m/n$: Müller ($2\log_2 n$) vs. Cogentic ($\bar{d} \geq 2000$), crossing at $\log_2 n = 1000$. **(b)** Remark 1.3 spanning-tree barrier: $d/2-1$ crosses $\log_2(d-1)$ at $d^* \approx 7.32$ and $\log_2 d$ at exact root $d = 8$.

2 Constructive Lean 4.35.0 Kernel

`lean/CogenticEdgeRecon.lean` compiles in 2.4 s under Lean 4.35.0 [1] with zero external imports, zero warnings, zero `sorry`, and zero `Classical.choice`. Auditing via `AuditAxioms.lean` confirms 5 zero-axiom theorems (`none`: #4, #6, #8, #10, #16) and 13 `[propext, Quot.sound]` theorems (`P,Q`; Table 1).

Proposition 1 (Constructive Lean 4 Kernel Scope)**.** All 18 statements in Table 1 compile constructively under Lean 4.35.0 without external libraries or classical axioms (`Classical.choice`). Notably, Theorem 3 proves $2 \cdot n! < n^n$ universally for all $n \geq 4$, Theorem 5 proves `leafCount` $t \leq$ `maxWeight` t by structural induction on arbitrary finite branching trees, Theorem 13 proves the exact `List Nat` pigeonhole-equality lemma underlying Lemma 4.3 band isolation, and Theorem 15 proves $(p-1)\,\mathrm{ord}_p(R!) \leq R$ universally for all $p \geq 2$ and truncation depths $s \in \mathbb{N}$.

3 Exact Zero-Float Certificate Auditor

The modules `edgerecon_exact.py` and `edgerecon_certifier.py` (under `src/`) use exclusively Python `int`, `fractions.Fraction`, and exact integer `math.comb/factorial` (statically audited via `ast.walk` for zero `float/complex` literals and zero floating-point imports). Raw JSON loading enforces strict duplicate-key rejection (`object_pairs_hook`), rejects floats and `NaN/Infinity` (`parse_float/parse_constant`), caps nesting depth at 16, and validates ASCII-only integer/rational strings.

Definition 1 (`EDGERECON-COGENTIC-CERT-1.0` Predicates)**.** A certificate with `schema_version="EDGERECON-COGENTIC-CERT-1.0"` is accepted iff every claimed invariant matches exact recomputation and its mode predicate holds:

- **Mode 1 (`edge_deck_hypomorphism_and_kelly`):** Validates simple graphs G, H ($4 \leq n \leq 10$, no loops/duplicates), computes exact $|\mathrm{Aut}(G)|$, $|\mathrm{Aut}(H)|$, exact edge-deck $\mathcal{D}_E$ and vertex-deck $\mathcal{D}$ multiset equality via 1-WL-pruned backtracking isomorphism, enforces verdict semantics (`NON_ISOMORPHIC_COUNTEREXAMPLE` only at $m \leq 3$, `ISOMORPHIC_DECK_MATCH` with verified bijection φ, or `DECK_MISMATCH`), checks Greenwell's isolate-free reduction, verifies exact $\mathbb{Z}$ edge-Kelly and vertex-Kelly sums across all listed patterns, and audits Müller's $2^{m-1} > n!$.
- **Mode 2 (`kruskal_multiscale_rigidity_cert`):** Verifies degree-monotone vertex order $\prec_V$, independently computes the unique lexicographic Kruskal MST $(T, \prec_V)$, audits at every scale Δ_k Lemma 3.1(1a) spanning forest on $G[B_k]$, (1b) $|B_{\Delta_k}| \leq |H_k| - 1$ and $|E(T_{\mathrm{core}}[\Delta_k])| \leq 2|H_k| - 2$, (1c) canonical parent $u(C) = \min_{\prec_V}(N_G(C) \cap H_k)$, exact 2^{H_k}-labeled class multiplicities $\tilde{w}_{\mathcal{T}}$, Lemma 3.1(2) subgroup product $\prod \tilde{w}_{\mathcal{T}}! \mid |\mathrm{Aut}(G)|$, and Lemma 6.1(1) exact drops $\Phi_\Delta(G) - \Phi_\Delta(G - e) = |e \cap H_k|$.
- **Mode 3 (`shift_invariant_charlier_band_cert`):** Parses sparse exact-$\mathbb{Q}$ homogeneous $P \in \mathbb{Q}[y_1, \ldots, y_M]$ of degree N_0, certifies exact shift-invariance $D_I(P) \equiv 0$, verifies exact ratio $\rho = \|P\|_1 / |[y^v]P|$ against Theorem 4.2 ($\rho^{200t} \geq (t+1)^{N_0}$ and $\rho^2 3^{N_0} \geq 4^{N_0}$ at t=1), verifies Lemma 4.1(1) exact rational Charlier bound $|I_{d,k}(r; 1/(2r))| \leq 1 - \frac{11d}{24r}$ for $0 \leq k \leq k_{\max}$, and optionally executes Lemma 4.3 band isolation $F_{\mathrm{mid}} = C_{\mathrm{const}}^{-1}[y_{I_{\mathrm{hi}}}^{\gamma_{\mathrm{hi}}}] H_w(P)$, certifying I_{lo}-independence, coefficient preservation, $\|F_{\mathrm{mid}}\|_1 \leq \|P\|_1$, and $D_{I_{\mathrm{mid}}}(F_{\mathrm{mid}}) \equiv 0$.
- **Mode 4 (`bridge_rigidity_and_density_audit`):** Computes the exact bridge restriction group $Q_D \leq \mathrm{Sym}(W_D \setminus S)$ of Lemma 5.1(2) ($|W_D \setminus S| \leq 8$), verifying every prime $p \mid |Q_D|$ satisfies $p < \Delta$ and $|Q_D| \leq (16\Delta)^R$; audits Theorem 1.1 regimes (`VACUOUS_SIMPLE_GRAPH_CAP`, `ACTIVE_LINEAR_DENSITY`, `BELOW_1000N_THRESHOLD`) and exact 2^{m_0-1} vs. $n_0^{n_0}$; and optionally audits Pillar III twin classes of size $\in \{1, 2\}$ with $r < \Delta_{k-1}$ and pairwise-distinct open neighborhoods ($\geq 3/8$ selected), half-degree preservation $2d_{K_0}(x) \geq d_G(x)$, and exact degree-excess conservation $\sum_{u \in H_{k-1}}(d_G(u) - d_{K_0}(u)) = r|W_0|$.

Table 1: Complete inventory of all 18 constructive Lean 4.35.0 theorems in `lean/CogenticEdgeRecon.lean` (zero `sorry`, zero `axiom`, zero `Classical.choice`, zero warnings; `none` for #4, #6, #8, #10, #16 and `[propext, Quot.sound]` (`P,Q`) for the remaining 13).

#	Theorem Identifier	Ref. [2]	Exact Machine-Checked Statement & Hypotheses	Ax.
1	`cor12_min_degree_implies_linear_density`	Cor. 1.2	$\forall ds : \texttt{List Nat}$, $\texttt{AllGe}\ 2000\ ds \implies 2000 \cdot ds.\texttt{length} \le \texttt{sumNat}\ ds$.	P,Q
2	`thm11_vacuous_small_n`	Thm. 1.1	$\forall n_0 \in \mathbb{N},\ 1 \le n_0 \wedge n_0 \le 2000 \implies n_0(n_0-1)/2 < 1000 n_0$.	P,Q
3	`lem22_muller_factorial_bound`	Lem. 2.2	$\forall n \ge 4,\ 2 \cdot n! < n^n$ (universal induction on $k = n-4$).	P,Q
4	`rem13_spanning_tree_barrier_at_d8`	Rem. 1.3	$2^3 = 8$, $2^{d-2} < d^2$ for $d \in \{3,4,5,6,7\}$, and $9^2 < 2^7$ (exact $\bar{d} = 8$ root).	none
5	`lem24_decision_tree_kraft_inductive`	Lem. 2.4	Structural induction on `DecTree`: $\forall t$, $\texttt{leafCount}\ t \le \texttt{maxWeight}\ t$.	P,Q
6	`lem24_numerical_e_ln2_pi_rational`	Lem. 2.4	$271{\cdot}60 < 163{\cdot}100$, $1631{\cdot}100 < 272{\cdot}600$, $693{\cdot}1215 < 842{\cdot}10^3$, $311{\cdot}648 < 2020{\cdot}100$, $168^4 < 8{\cdot}10^8$, $4^3 > 2{\cdot}3^3$, $2^{10} > 4{\cdot}3^5$.	none
7	`lem31_bridge_count_bound`	Lem. 3.1	$1 \le h$, $e_{\text{core}} = h + b - 1$, $2b \le e_{\text{core}} \implies b \le h - 1 \wedge e_{\text{core}} \le 2h - 2$.	P,Q
8	`lem31_nonbridge_entropy_margin`	Lem. 3.1	$1000{-}252{-}10{-}160 = 578$, $578{-}250{-}20 = 308 > 150$, $47{\cdot}100 < 16{\cdot}300$, $300{\cdot}272 < 100{\cdot}1024$.	none
9	`lem33_dyadic_sum_geometric`	Lem. 3.3	$\forall d_0 \ge 1,\ r \in \mathbb{N}$, $\texttt{dyadicSum}\ d_0\ r + d_0 = 2(d_0 2^r) \wedge \texttt{dyadicSum}\ d_0\ r < 2(d_0 2^r)$.	P,Q
10	`lem33_tail_error_10_neg_34`	Lem. 3.3	$4 \cdot 127 < 2^9$, $10^3 < 2^{10}$, and $10^{34} < 2^{116}$ (exact tail-error domination).	none
11	`lem42_t1_charlier_ratio_decay`	Thm. 4.2	$\forall k \ge 2,\ (2k+1)^2 < 3(2k-1)^2$ (certifying $(2k+1)/((2k-1)\sqrt{3}) < 1$).	P,Q
12	`lem42_charlier_duality_2var`	Thm. 4.2	$2c_0 + c_1 = 0 \wedge c_1 + 2c_2 = 0 \implies c_1 = -2c_0$, $c_2 = c_0$, annihilates all $au_1 + bu_2 + d$.	P,Q
13	`lem43_band_localization_partition`	Lem. 4.3	Pointwise $u \ge w$ with equal sums $\sum u = \sum w \implies u = w$ (exact `List Nat` induction).	P,Q
14	`lem51_chebyshev_primorial_step`	Lem. 5.1	$\forall k \in \mathbb{N},\ 4^{k+1} \cdot 4^k = 4^{2k+1} \wedge 6^4 < 12^3$.	P,Q
15	`lem51_legendre_valuation_bound`	Lem. 5.1	$\forall p \ge 2,\ s, R \in \mathbb{N},\ (p-1) \cdot \sum_{j=1}^{s} \lfloor R/p^j \rfloor \le R$ (universal induction).	P,Q
16	`prop84_chebyshev_sampling_3_8`	Prop. 8.4	$2 \cdot 8 - 1 = 3 \cdot 5$ ($2/5 - 1/40 = 3/8$), $384 \cdot 40 < 40000$, $800 < 3 \cdot 92 \cdot 3$.	none
17	`prop84_degree_excess_conservation`	Prop. 8.4	$\forall r, n_W \in \mathbb{N},\ \sum \text{replicate}(n_W, r) = r n_W$ and $10^6 < 8 \cdot 200 \cdot 693$.	P,Q
18	`sec9_final_contradiction_closure`	Sec. 9	$m \ge 4000$, $15m{\cdot}10^{32} < (S_1{+}S_2)10^{34}$, $S_1, E_3, W_{\text{sum}} < m/10^{34}$, $S_2 \le \frac{m}{10^{34}} + E_3 + W_{\text{sum}} \implies \bot$.	P,Q

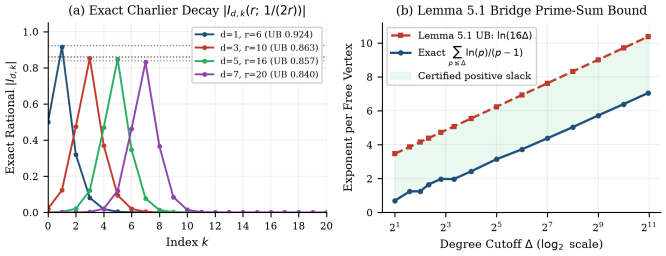

Figure 2: **(a)** Exact rational Charlier profiles $|I_{d,k}|$ peaking at $k = d$ strictly below Lemma 4.1(1)'s bound $1 - 11d/(24r)$. **(b)** Lemma 5.1 prime sum $\sum_{p \le \Delta} \ln(p)/(p-1)$ vs. $\ln(16\Delta)$ across $\Delta \in [2, 2000]$.

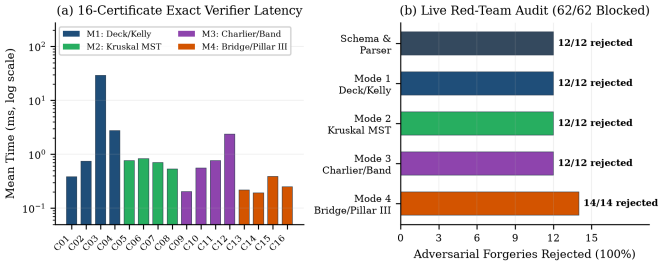

Figure 3: **(a)** Mean zero-float verification latency across all 16 canonical certificates (0.192–28.989 ms, log scale). **(b)** Live adversarial red-team audit rejecting 62/62 forgeries across five attack classes with exact error-branch assertions.

4 Exact Findings, Worked Witnesses, and Audit

All experiments ran locally on Windows 11 (Python 3.12.6, Lean 4.35.0; 7-run means after warm-up).

1. Linear-Density Crossover, Remark 1.3 Barrier & Charlier/Bridge Bounds (Figs. 1–2). Theorem 1.1 is vacuous for $n_0 \le 2000$ ($\binom{2000}{2} = 1{,}999{,}000 < 2{,}000{,}000$), activates at $n_0 = 2001$ (K_{2001} has exact $m_0 = 2{,}001{,}000$), and strictly beats Müller's $n_0 \log_2 n_0$ for all $\log_2 n_0 > 1000$—certified verbatim at $n_0 = 2^{1005}, m_0 = 1000\, n_0$ (`C16`, where $2^{m_0 - 1} < n_0^{n_0}$ yet Theorem 1.1 holds). Exact rational Charlier profiles (Fig. 2a) confirm tight positive margins across $(d, r) \in \{(1,6), (2,8), (3,10), (4,12), (5,16), (7,20)\}$, e.g., at $(d, r) = (1, 6)$ peaking at $|I_{1,1}| = 11/12 = 0.91667 < 133/144 = 0.92361$, and at $(7, 20)$ peaking at $|I_{7,7}| = 0.83147 < 403/480 = 0.83958$. The Lemma 5.1 prime sum grows from 0.69315 ($\Delta = 2$, slack 2.77259) to $7.04814 < \ln 32000 = 10.37349$ ($\Delta = 2000$, slack 3.32535).

2. Hand-Verifiable Worked Exact Witnesses (One per Mode). To ensure complete standalone auditability directly from this manuscript:

- **Mode 1 (`C02`, m=3 Harary pair):** $G = K_3 \sqcup K_1$ ($E_G = \{(0,1), (1,2), (0,2)\}$, degs $(0,2,2,2)$, $|\text{Aut}| = 6$) vs. $H = K_{1,3}$ ($E_H = \{(0,1), (0,2), (0,3)\}$, degs $(1,1,1,3)$, $|\text{Aut}| = 6$). Deleting any edge yields $K_{1,2} \sqcup K_1$, so $\mathcal{D}_E(G) = \mathcal{D}_E(H)$ while $\mathcal{D}(G) \ne \mathcal{D}(H)$; edge-Kelly gives $(3-1) \cdot s(K_2, G) = 6 = \sum_e 2$ and $(3-2) \cdot s(P_3, G) = 3 = \sum_e 1$.
- **Mode 2 (`C05`, n=12, m=15):** Triangle hubs $\{0,1,2\}$ (all deg 5), three twin K_2-pendants $\{(3,4), (5,6), (7,8)\}$ attached to hub 0, and three degree-2 twins $\{9,10,11\}$ adjacent to $\{1,2\}$. At $\Delta_0 = 3$ ($H_0 = \{0,1,2\}$), Kruskal selects $\{9\}$ as the sole bridge ($1 \le |H_0| - 1 = 2$, core edges $3 \le 4$), leaving 3 twin K_2-pendants on hub 0 (3!) and 2 non-bridge $\{1,2\}$-twins (2!), certifying $3! \cdot 2! = 12 \mid 72 = |\text{Aut}(G)|$, with 3 internal Φ_3-zero-drop edges.
- **Mode 3 (`C11`, 4-var Vandermonde):** $P = \prod_{0 \le i < j \le 3} (y_i - y_j)$ has $N_0 = 6$, 24 terms (± 1), $D_I(P) \equiv$

Table 2: Canonical 16-certificate `EDGERECON-COGENTIC-CERT-1.0` atlas (`results/exp1_certificate_atlas.json`; all 16 verified).

Certificate ID	M	Exact Certified Witness Summary	ms
`m1_harary_m2_counterexample`	M1	$2K_2$ vs. $K_{1,2} \sqcup K_1$: $\mathcal{D}_E{=},\mathcal{D}_{\neq}$, $\lvert\mathrm{Aut}\rvert{=}8,2$	0.384
`m1_harary_m3_counterexample`	M1	$K_3 \sqcup K_1$ vs. $K_{1,3}$: $\mathcal{D}_E{=},\mathcal{D}_{\neq}$, $\lvert\mathrm{Aut}\rvert{=}6,6$	0.743
`m1_muller_dense_n6_iso_kelly`	M1	$n{=}6, m{=}11$: $2^{10} > 6!$, 5 Kelly $\mathbb{Z}$-ids ok	28.989
`m1_harary_m5_deck_distinguished`	M1	C_5 vs. lollipop ($m{=}5$): $\mathcal{D}_E \neq$, 4 Kelly ok	2.730
`m2_kruskal_n12_hub_twins_exact_aut`	M2	$n{=}12, m{=}15$: $\prod \bar{w}!{=}12 \mid 72$, Φ_3: 3 int.	0.761
`m2_kruskal_n14_two_tier_dyadic`	M2	$n{=}14, m{=}17$: 2 scales, $12 \mid 96$, Φ_3: 4 int.	0.827
`m2_kruskal_n24_three_tier_dyadic`	M2	$n{=}24, m{=}31$: 3 scales, $\prod \bar{w}!{=}1728$, Φ_6 ok	0.697
`m2_kruskal_n20_dual_cliques_bridges`	M2	$n{=}20, m{=}32$: 2 scales, $\prod \bar{w}!{=}576$, Φ_3 ok	0.532
`m3_shift_t1_multilinear_n2`	M3	$M{=}4, N_0{=}2, t{=}1$: $\rho{=}4$, $\lvert I_{1,1}\rvert{=}11/12$	0.204
`m3_shift_t1_multilinear_n4`	M3	$M{=}8, N_0{=}4, t{=}1$: $\rho{=}16$, $\lvert I_{3,3}\rvert < 69/80$	0.558
`m3_vandermonde4_lemma43_band`	M3	V_4 ($N_0{=}6$): band $[1,2] \to$ deg-1, $\lVert F_{\mathrm{mid}}\rVert_1{=}2$	0.763
`m3_vandermonde5_lemma43_band`	M3	V_5 ($N_0{=}10$): band $[1,3] \to$ deg-3, $\lVert F_{\mathrm{mid}}\rVert_1{=}6$	2.362
`m4_bridge_d4_wreath48_vacuous2000`	M4	$\Delta{=}4, R{=}6, \lvert Q_D\rvert{=}48$; $n_0{=}2000$ vacuous	0.218
`m4_bridge_d5_pillar3_r2_active2001`	M4	$\Delta{=}5, \lvert Q_D\rvert{=}6$; P3 $r{=}2$, exc. 8; $n_0{=}2001$	0.192
`m4_bridge_d6_pillar3_r3_active1e6`	M4	$\Delta{=}6, \lvert Q_D\rvert{=}24$; P3 $r{=}3$, exc. 12; $n_0{=}10^6$	0.387
`m4_bridge_d6_beyond_muller_2pow1005`	M4	$\Delta{=}6, \lvert Q_D\rvert{=}6$; $n_0{=}2^{1005}$ beats Müller!	0.249

0, target $v = (3,2,1,0)$, $\|P\|_1/|c_v| = 24$. Band $[t_0,t_1] = [1,2]$ gives $I_{\mathrm{lo}} = \{3\}$, $I_{\mathrm{mid}} = \{1,2\}$, $I_{\mathrm{hi}} = \{0\}$, $C_{\mathrm{const}} = \binom{3}{1}\binom{2}{1}\binom{1}{1} = 6$, and $F_{\mathrm{mid}}(y_1,y_2) = y_1 - y_2$ ($\|F_{\mathrm{mid}}\|_1 = 2 \le 24$, $D_{I_{\mathrm{mid}}}(F_{\mathrm{mid}}) \equiv 0$).

- **Mode 4 (`C13` & `C14`):** In `C13`, K_6 hubs $\{0..5\}$ attach to $D = \{6,7,8,9\} \subset V_{<4}$ (root $v_0 = 6$, leaves $7 \to \{0,1\}, 8 \to \{2,3\}, 9 \to \{4,5\}$); $Q_D \cong S_2 \wr S_3$ has exact order $48 = 2^4 \cdot 3$ (primes $\{2,3\} < 4$, $48 \le 64^6$). In `C14` ($\Delta = 5$, K_6 hubs + 4 degree-2 twin classes $\{[11,12],[13,14],[15],[16,17]\}$), selecting 2/4 classes ($\ge 3/8$) gives $W_0 = \{11,12,13,14\}$, preserves $2d_{K_0}(x) \ge d_G(x)$, and satisfies $\sum_{u \in H_5}(d_G(u) - d_{K_0}(u)) = 8 = 2 \cdot |W_0|$.

3. 16-Certificate Atlas & 66-Test Red-Team Audit (Table 2, Fig. 3). Table 2 summarizes all 16 canonical certificates (0.192–28.989 ms). `tests/test_suite.py` runs 66 pytest functions: 1 full replay of all 16 certificates, 3 raw-JSON parser guards (IEEE-754 float, `NaN`, duplicate keys), and 62 parametrized live adversarial forgeries (12 schema/parser + 12 each for Modes 1–3 + 14 for Mode 4, covering self-loops, non-monotone Kruskal orderings, forged MST edges, non-shift-invariant polynomials, non-uniform Lemma 4.3 partitions, disconnected bridge subsets, broken open-twin classes, $r \ge \Delta_{k-1}$ leaf-stratum violations, split size-4 twin classes, $< 3/8$ sampling ratios, and half-degree violations), each asserting an exact regex match on its intended `CertificateError` branch. `scripts/verify_all.py` enforces all 5 gates including strict axiom whitelisting and a 40-file SHA-256 manifest.

5 Conclusion

We built an independent, explicitly scoped, zero-floating-point verification stack for Google Research Cogentic's unconditional linear-density edge reconstruction theorem ($m \ge 1000\,n_0$) [2], uniting 18 constructive Lean 4.35.0 theorems (zero `sorry`, zero `Classical.choice`), the pure-`int`/`Fraction` 4-mode auditor `EDGERECON-COGENTIC-CERT-1.0`, 16 canonical exact witnesses spanning Harary's $m \le 3$ boundary to $n_0 = 2^{1005}$ beyond Müller, and 62/62 live adversarial forgery rejections.

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