The Eriksson Programme (Yang–Mills): Auditable Claim-to-Theorem Manifest, Certified Obstructions, and Continuation Frontier (Release `yang-mills-archive-v1.0`)

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23 November 2026 (v2)

Abstract

This archival technical note provides a self-contained, auditable claim-to-theorem specification and continuation handoff for the Yang–Mills Lean 4 formalization inside `THE-ERIKSSON-PROGRAMME` at archival release `yang-mills-archive-v1.0` (commit `6b54eca695e96b3038ceda18276c4e28fb45983f`; canonical Lean core checkpoint `02fae0ad` in `project-state.json`, Lean `v4.29.0-rc6`, pinned `mathlib` `07642720480157414db592fa85b626dafb71355b`). Responding to every material objection raised against v1 (`ARR-2026-7DYZAEKGJP8FPTJ1` v1), this v2 revision replaces directory-level summaries with exact fully-qualified Lean 4 theorem statements, explicit coupling and geometric hypotheses, concrete strictly-interacting non-vacuity witnesses, exact operator classes and volume quantifiers for the certified obstructions, a strict distinction between `YangMillsCore`-compiled, `oracle_check`-audited, and standalone declarations, and resolved AIRR identifiers for all 32 indexed mathematical-physics records.

We state the mathematical boundary without pseudo-metrics. The verified core proves fixed-spacing lattice results: a Kotecký–Preiss / Mayer–Ursell polymer engine; volume-uniform strong-coupling SU(N_c) Wilson-loop area laws (indexed by $\mathbb{Z}/N_c\mathbb{Z}$-chain homology area `chainAreaA`) and exponential clustering of truncated two-plaquette correlations without Peter–Weyl; an explicit strictly-interacting SU(2) non-vacuity window $0 < |\beta| \leq 10^{-5}$; exact $\mathbb{Z}_n$ centre-symmetry selection rules for positively-oriented Wilson loops with $n \nmid |es|$; conditional 4D marginal-coupling UV-summation theorems requiring $\kappa_0 > 1$; real-order Amos/Bessel ratio bounds on $\nu \geq 0$; and a transfer-matrix identification chain for the 1D open $\mathbb{Z}_2$ Ising spin chain. No four-dimensional continuum limit ($a \to 0$), no physical SU(N) Osterwalder–Schrader or Wightman reconstruction, and no solution to the Clay Millennium Yang–Mills Mass Gap problem is claimed. We separate open formalization tasks (fixed-spacing SU(N) Wilson reflection positivity and the Balaban/Dimock `hRpoly` estimate in draft PR #29) from open mathematics (4D continuum Yang–Mills existence and mass gap).

1 Verification Architecture and Coverage Boundary

To eliminate ambiguity between built modules, audited declarations, and standalone work, the archival release `yang-mills-archive-v1.0` maintains three distinct automated boundaries:

1. **Core compilation and axiom oracle (`lean-build.yml`).** On every push and PR to `main` (with a cache-free `cold_build: true` manual dispatch option), GitHub Actions installs Lean `v4.29.0-rc6`, fetches prebuilt `mathlib .olean` caches for commit `07642720480157414db592fa85b626dafb71355b`, runs `lake build YangMillsCore`

(recorded core count: 8466 jobs in `project-state.json`), builds the modules imported by `oracle_check.lean`, executes `lake env lean oracle_check.lean`, and verifies via `scripts/check_oracle_output.py` that every `#print axioms` readout depends only on `[propext, Classical.choice, Quot.sound]`.

2. **Lexical honesty guard (`ci.yml`).** `scripts/check_consistency.py` scans `YangMills/` and `Lean/` (excluding `YangMills/Experimental/`) for `sorry`, and scans the verified-core directories (`L0_Lattice`, `L1_GibbsMeasure`, `KP`, `ClayCore`, `Paper`, `RG`, `YangMillsCore.lean`, and core `P8_PhysicalGap` Haar/compactness files) for `axiom`. A lexical scan alone never certifies compilation or non-vacuity; it is strictly supplementary to `lean-build.yml` and the non-vacuity witnesses below.

3. **Standalone and branch-only modules (explicitly outside `YangMillsCore`).** The finite SU(2) Wilson cylinder geometry/measure modules (R1/R2, Sept 2026) and draft PR #29 (`hRpoly`) are *not* imported by `YangMillsCore.lean` and are *not* covered by the core build guarantee.

2 Auditable Claim-to-Theorem Manifest

Every headline claim in this archive maps to a concrete fully-qualified Lean 4 declaration in `YangMillsCore` (at commit `6b54eca695e96b3038ceda18276c4e28fb45983f`). Below we state the decisive domains, quantifiers, coupling windows, and non-vacuity witnesses so each statement can be checked directly from this note.

2.1 KP / Mayer polymer engine (`YangMills/KP/`)

- `YangMills.KP.partition_eq_exp_clusterSum_of_kp` (`KP/MayerInversion.lean:2038`): for any finite polymer model P (`[Fintype P.Polymer]`) and weight function $a : P.\mathrm{Polymer} \to \mathbb{R}$ satisfying the Kotecký–Preiss criterion `KPCriterion P a`, the polymer partition function satisfies $\Xi = \exp(\mathrm{clusterSum}\ P)$ with absolutely summable cluster series.

- `YangMills.KP.abs_ursell_le_card_spanningTrees` and `kp_convergence`: proves Penrose's tree-graph bound $|\phi(X)| \le \#\mathrm{SpanningTrees}(G_X)$ via greedy BFS partition schemes and yields exponential convergence under the KP smallness condition.

2.2 Strong-coupling $\mathrm{SU}(N_c)$ area law, IR clustering, and non-vacuity (`YangMills/L1_GibbsMeasure/`)

- **Volume-uniform exact-action Wilson-loop area law:** `YangMills.normalized_exp_wilson_loop_area_law` (`L1_GibbsMeasure/RestrictedGate.lean:2045`). For any dimension d, lattice side N, gauge rank $N_c \ge 1$ (`[NeZero N_c]`), arbitrary edge list es : List (ConcreteEdge d N), complex plaquette couplings $\|c_p\| \le \delta$ with $c'_p = \overline{c_p}$, and parameters $t, \varepsilon \ge 0$, $\sigma \in [0,1]$ satisfying the explicit volume-independent KP smallness inequalities

$$(16d+1)^2(e^{2\delta N_c}-1)e^{t+\varepsilon+1} < 1, \quad (16d)\frac{(e^{2\delta N_c}-1)e^{t+\varepsilon+1}}{1-(16d+1)^2(e^{2\delta N_c}-1)e^{t+\varepsilon+1}} \le t,$$

together with $(16d+1)^2\sigma < 1$ and the effective-activity bound $(e^{2\delta N_c}-1)\exp(\cdots) \le \sigma^2$, the normalized expectation of $\mathrm{tr}(U_{es})$ under the exact exponential plaquette action

$\prod_p \exp(c_p \operatorname{tr} U_p + \overline{c_p}\,\overline{\operatorname{tr} U_p})$ obeys

$$|\langle \operatorname{tr} U_{es} \rangle| \;\le\; N_c \, \exp\!\big(4d\,|\mathrm{edgeSupport}(es)|\, C_{\mathrm{perim}}\big)\, \sigma^{\,\mathrm{chainAreaA}_{\mathbb{Z}/N_c\mathbb{Z}}(\mathrm{loopChain}(es))},$$

with constants independent of lattice side N. Because the exponent is the minimal spanning-surface area in $\mathbb{Z}/N_c\mathbb{Z}$-chain homology (`chainAreaA`), twice-traversed SU(2) loops have $\mathrm{loopChain}_{\mathbb{Z}/2\mathbb{Z}} = 0$ and hence chainAreaA $= 0$ ($\sigma^0 = 1$), cleanly accommodating the exact Haar identity $\mathbb{E}[\mathrm{tr}(U_C^2)/2] = -1/2$ at $\beta = 0$.

- **Unconditional $\mathrm{SU}(N_c)$ two-plaquette exponential clustering without Peter–Weyl:** `YangMills.sun_two_plaquette_correlator_bound` (`L1_GibbsMeasure/TwoPlaquetteCorrelator.lean:710`). For any $N_c \ge 1$, any measurable $f : \mathrm{SU}(N_c) \to \mathbb{R}$ with $|f| \le 1$, any $s > 0$, $\beta \in \mathbb{R}$, $t, \varepsilon \ge 0$ satisfying $(16d+1)^2\big((e^{|\beta|N_c} - 1) + s + (e^{|\beta|N_c} - 1)s\big)e^{t+\varepsilon+1} < 1$ and its paired KP smallness bound, and any two distinct plaquettes $p \ne q$ at touching-graph distance $2k \le \mathrm{dist}_{\mathrm{touchGraph}}(p, q)$, the connected two-plaquette correlator under the Wilson Gibbs measure with action $-\beta\,\mathrm{wilsonAction}(\mathrm{Re\,tr})$ decays as $|\mathrm{Cov}_\beta(f(U_p), f(U_q))| \le C(d, N_c, \beta, s, t, \varepsilon)\, e^{-\varepsilon k}$, uniformly in lattice side N.

- **Explicit strictly-interacting non-vacuity witness:** `YangMills/L1_GibbsMeasure/ThermodynamicNonvacuity.lean` proves that the uniform local KP regime is non-empty for the strictly interacting SU(2) Wilson theory with nonconstant plaquette energy $\mathrm{Re\,tr}\, U$ on the explicit interval $0 < |\beta| \le 10^{-5}$ with $t = \varepsilon = \eta = 10^{-2}$ (and explicitly rules out degeneration to the free $\beta = 0$ point).

- **Fixed-spacing thermodynamic limit:** `YangMills/L1_GibbsMeasure/IntegerThermodynamicLimit.lean` constructs a translation-invariant infinite-volume state on finite-support integer-lattice edge observables (`IntegerLocalObservable`) along periodic torus exhaustions of side $n + 1 \to \infty$ at fixed spacing inside `UniformLocalKPRegime`, passing two-plaquette clustering to the limit (`ThermodynamicCorrelation.lean`). This is an infinite-volume lattice limit, not an ultraviolet continuum limit.

- **Exact $\mathbb{Z}_n$ centre-symmetry selection rule:** `YangMills.integral_wilsonLoopSU_gibbs_eq_zero` (`L1_GibbsMeasure/GibbsSelectionRule.lean:120`). Under the global centre action multiplying every positively-oriented edge by $\omega = e^{2\pi i/n} I_n \in Z(\mathrm{SU}(n))$, every plaquette traverses two forward and two backward edges so the Wilson action is exactly invariant for all $\beta \in \mathbb{R}$. Any positively-oriented edge list es (`hpos`) transforms with eigenvalue $\omega^{|es|}$, so its interacting Gibbs expectation vanishes whenever $n \nmid |es|$. (Contractible plaquettes have signed net charge $0 \pmod n$ and are not forced to vanish.)

2.3 Conditional 4D marginal-coupling RG assembly (`YangMills/RG/`)

- `YangMills.RG.lattice_mass_gap_of_singleScaleUVDecay_marginal` (`RG/MarginalUVMassGap.lean:132`): an abstract real-sequence theorem taking sequences $\mathrm{covIR}, g : \mathbb{N} \to \mathbb{R}$ and per-scale remainders $R_{\mathrm{sc}} : \mathbb{N} \to \mathbb{N} \to \mathbb{R}$ with $g_k > 0$, $\beta g_k < 1$, exact one-loop logistic recursion $g_{k+1} = g_k(1 - \beta g_k)$, exponent $\kappa_0 > 1$ (guaranteeing absolute summability $\sum_k g_k^{\kappa_0} < \infty$, unlike $\kappa_0 = 1$ where $O(1/k)$ diverges), exponential IR decay $|\mathrm{covIR}_k| \le C_1 e^{-\varepsilon k}$, and carried UV hypothesis `SingleScaleUVDecay Rsc g C2 c0 kappa0`. It concludes exponential decay of $\mathrm{covIR}_t + \mathrm{covUV}_t$. It mentions no gauge group, Hilbert space, or transfer matrix; the missing physical input is `hRpoly`.

2.4 Modified Bessel / Amos suite and 1D $\mathbb{Z}_2$ transfer chain

- **Real-order Amos bounds (`AmosClosure/AmosBoundProofReal.lean`):** for all real orders $\nu \geq 0$ and $x > 0$ (starting from geometric series domination on $0 < x \leq 1/4$ and extending via Riccati barrier lemmas), proves the strict Amos-type inequality for $I_{\nu+1}(x)/I_\nu(x)$, companion crossing classifications (`ARR-2026-39Y0F83F4Z9JK90G`), and global ratio monotonicity of the killed von Mises bridge (`ARR-2026-3CEWH763KG84B8TM`).

- **1D open $\mathbb{Z}_2$ Ising transfer-matrix chain (`YangMills/OS/Z2Identification.lean`):** proves $\mathbb{E}_n[\overline{A(\sigma_0)}B(\sigma_n)] = \langle A\Omega, T_\beta^n(B\Omega)\rangle_{\mathbb{C}}$, exact correlation $(\tanh\beta)^n$, spectral gap $1 - |\tanh\beta|$ ($1 - \tanh\beta$ for $\beta \geq 0$), and time-zero pairing non-degeneracy (`ARR-2026-7PRDH0DHH280HRKH` v2). It is a 1D finite-slice spin chain, not a 4D SU(N) OS reconstruction; strictly positive kernel entries alone do not imply reflection positivity, and finite-extent Perron isolation does not imply a uniform gap.

3 Certified Negative Results and Exact Obstruction Classes

1. **Volume-uniform Poincaré walls (`ARR-2026-3Q0VH7PMNV8C9VQE`):** for the flat gauge-fixed Hodge plus block-averaging quadratic form $\langle A, K_0 A\rangle + a\|QA\|^2$ on the fine d-dimensional torus of side LN' ($d \geq 3$, $N_c \geq 2$, concrete $\mathfrak{su}(N_c)$ adjoint model), every flat block-Poincaré constant satisfies $C_P \geq L^{d-2}$, refuting any volume-uniform coercivity constant via this route. In the fluctuation sector on fine side $(2M)N'$, transported half-period square-wave modes satisfy $\|QA\|^2 \leq (2M)^{-1}\|A\|^2$ and $\langle A, K_0 A\rangle = 8((2M)N')^{-1}\|A\|^2$, forcing $C_P \geq 2M/9$ for every $N' \geq 1$, $d \geq 3$, $N_c \geq 2$, and every adjoint model.

2. **Flat lattice Hodge factorized-penalty and coarse-rank walls** (`ARR-2026-627PF1B4KK9VTRYT`, `ARR-2026-7EPM3V3DP796BBRR`, and expanded v2 dossier `papers/flat-coarse-rank/`): on flat periodic tori, divergence-zero exact-kernel Fourier families annihilated by the specified block-averaging map Q obstruct every penalty of the form Q^*MQ factored through Q.

3. **Spatial coupling and Birkhoff obstructions in $\mathbb{Z}_2$ slices** (`ARR-2026-4X084GVAYF87PB3F`, `ARR-2026-6M8PMVHTMH8YZ9ND`, `ARR-2026-1A608PTGTC9XX9JZ`): isolates where uniform-vacuum and extent-independent bounds fail under spatial coupling.

4. **Parity barriers for decoupling inequalities (`ARR-2026-5HRC3PS9HZ9WDAA5`):** no comparison functional of bounded marginal order can certify uniform decoupling.

5. **Sine-series Wronskian counterexamples (`ARR-2026-5FKM2GGK348CPTE9` v2):** explicit finite arrays $a = (3,4,3), b = (3,2,1)$ ($W(2\pi/3) = 3\sqrt{3} > 0$) and $a = (6,10,24), b = (6,5,2)$ ($W(\pi - \varepsilon) = -8\varepsilon^3 + O(\varepsilon^5) < 0$ yet $W(2\pi/3) = 51\sqrt{3} > 0$) refute unrestricted sine-series Biernacki–Krzyż and generic local-to-global lifting.

6. **Two-transporter Haar projection no-go (`docs/SU2-TWO-TRANSPORTER-NOGO-20260731.md`):** two free Haar-integrated transporters absent from the observable project the reduced SU(2) quadratic form to the trivial sector.

4 Open Formalization Tasks vs. Open Mathematics

We separate bounded formalization tasks (classically known mathematics not yet formalized in this Lean archive) from genuinely open mathematics:

1. **Open formalization task — physical $\mathrm{SU}(N)$ Wilson reflection positivity at fixed spacing:** classically proved by Osterwalder and Seiler (1978), but still open in this Lean archive beyond the standalone R1/R2 finite SU(2) cylinder geometry and normalized product-Haar measure (`docs/evidence/physical-cylinder-r2-20260906/`, outside `YangMillsCore`).

2. **Open formalization / constructive-QFT task — `hRpoly` (frozen in draft PR #29):** concrete Balaban/Dimock cluster-expansion-with-holes remainder activity decay for the physical gauge RG operator. Frozen checkpoint (`docs/HRPOLY-STATUS.md`): 20/41 terminal `PreEq136` producers, TermSource $= 0$, uniform physical (B_0, δ_0) and regional transport (F5) open.

3. **Open mathematics — 4D continuum limit ($a \to 0$) and OS/Wightman reconstruction with positive mass gap for $\mathrm{SU}(N)$ ($N \geq 2$):** no complete proof exists in the literature or in this repository; removing the UV regulator while retaining a non-perturbative mass gap remains the Clay Millennium Problem.

5 Resolved AIRR Preprint Index (All 32 Records)

Table 1 maps all 32 mathematical-physics folders in `THE-ERIKSSON-PROGRAMME` to their live AIRR identifiers on https://airr.science/ (resolving the two entries that were pending in v1). Unrelated empirical audit manuscripts added under `papers/` in November 2026 are excluded.

Repository Folder	AIRR Identifier	Topic / Role
`paper/area-law`	`ARR-2026-2ZQ6F57N2H8MG91V`	Volume-uniform Wilson-loop area law
`papers/c1-rooted-tree-majorants`	`ARR-2026-505NQZHR7P9TQ8SA`	Rooted-tree majorants for polymers with holes
`papers/c2-amos-closure`	`ARR-2026-6DP8SDRZ678H9AY9`	Amos Bessel-ratio calculus across three sites
`papers/c3-amos-proof`	`ARR-2026-17K7KRR6NH9ZVAEJ`	Machine-checked Amos bound (integer order)
`papers/c4-amos-real`	`ARR-2026-3AJ7W5BS3N9VR8KM`	Machine-checked Amos-type bound (real order)
`papers/c5-crossing`	`ARR-2026-39Y0F83F4Z9JK90G`	Diagonal Amos-type crossing classification
`papers/c6-rg-interface`	`ARR-2026-0WG9YDY5BA9S3RF4`	Non-circular RG interface for Wilson correlators
`papers/bessel-amos-fh`	`ARR-2026-5PXDYNR2ER89HTQG`	Order-monotonicity of $(\log I_\nu)'$ & 2D LGT
`papers/phi-lemma`	`ARR-2026-72G1SSCNNG8YDVC2`	Weighted Turán-type Bessel monotonicity
`papers/surface-theorem`	`ARR-2026-1FPFA1TSMY9M3T9S`	Killed von Mises bridge ratio monotonicity
`papers/surface-complete`	`ARR-2026-3CEWH763KG84B8TM`	Global ratio monotonicity (closure)
`papers/wronskian-reduction`	`ARR-2026-5FKM2GGK348CPTE9`	Sine-series Wronskian, $F_B > 0$ proof & counterexamples
`papers/poincare-wall`	`ARR-2026-3Q0VH7PMNV8C9VQE`	Volume-uniform Poincaré walls
`papers/flat-coarse-rank`	`ARR-2026-7EPM3V3DP796BBRR`	Flat Hodge factorized-penalty wall (v2 dossier)

Repository Folder	AIRR Identifier	Topic / Role
papers/parity-barriers	ARR-2026-5HRC3PS9HZ9WDAA5	Parity barriers for decoupling inequalities
papers/o-bridge	ARR-2026-31ED4MTKC18JPSDE	Dense-family clustering-to-gap criterion
papers/reflection-positivity	ARR-2026-10JJK0VCCS87VTQH	$\mathbb{Z}_N$ reflection positivity ($\mathbb{Z}_2$ instance)
papers/os-chain-z2	ARR-2026-7PRDH0DHH280HRKH	Open $\mathbb{Z}_2$ Ising transfer-matrix chain
papers/os-quotient	ARR-2026-2TBRDN2DE598KSXD	Degenerate reflection pairing & GNS quotient
papers/perron-kernel	ARR-2026-2YDE42KFDY8Y3AA0	Perron theorem for strictly positive kernels
papers/perron-gap	ARR-2026-1A608PTGTC9XX9JZ	Finite-extent spectral gap (not uniform)
papers/spatial-extent	ARR-2026-4X084GVAYF87PB3F	Spatial coupling breaks uniform vacuum
papers/spatial-birkhoff	ARR-2026-6M8PMVHTMH8YZ9ND	Second spatial-extent obstruction
papers/spatial-gibbs	ARR-2026-2TTA0WSQBT89X964	Transfer bridge for spatial $\mathbb{Z}_2$ slice
papers/spatial-reflection	ARR-2026-02YRWX53DG9M3R0R	Endpoint reflection positivity ($\mathbb{Z}_2$ slice)
papers/spatial-os	ARR-2026-223XBWQ8PQ8Z2RDS	Bond reflection positivity & site form
papers/spatial-spectral	ARR-2026-1A7WS4SDF0894BPS	Fluctuation-sector operator bound
papers/spatial-uniform	ARR-2026-3EXXMXJD119J1R6R	Extent-independent rate (decoupled slice)
papers/congruence-spectrum	ARR-2026-7S1XA3PZEB9QM8FQ	Congruence rigidity and the fusion bound
papers/su2-crossing-compensated	ARR-2026-5V5K6T6DQT9J8SY6	Compensated-flow SU(2) crossing Ward
papers/thermodynamic-limit-kp	ARR-2026-5SGNYDASRB9V9VF9	Thermodynamic limit for local Gibbs states
papers/stationary-doob347	ARR-2026-1AS15VP0199VZ847	Boolean Doob kernels: stationary marginals
papers/stationary-weighting354	ARR-2026-4T35VQ6MZB8F99F4	Boolean Doob kernels: moment transfer

Table 1: Complete cross-reference of Yang–Mills and mathematical-physics manuscripts in THE-ERIKSSON-PROGRAMME to live AIRR archive identifiers.

6 Reproducibility, Citation, and Disclosure

Reproduce the verified Lean 4 core at release `yang-mills-archive-v1.0` (commit `6b54eca695e96b3038ceda18276c4e28fb45983f`; on Windows pass `-c core.longpaths=true`):

```
git clone -c core.longpaths=true \
  https://github.com/lluiseriksson/THE-ERIKSSON-PROGRAMME.git
cd THE-ERIKSSON-PROGRAMME && git checkout yang-mills-archive-v1.0
lake exe cache get && lake build YangMillsCore
lake env lean oracle_check.lean && python scripts/check_consistency.py
```

Code is licensed AGPL-3.0-or-later; citation metadata live in `CITATION.cff` and `.zenodo.json`. Large language models assisted formalization and editing under author supervision; core claims are checked by the Lean 4 kernel with standard axioms only. Author Lluis Eriksson administers AIRR (https://airr.science/); no independent editorial admission or peer review is claimed.