The Eriksson Programme (Yang–Mills): Machine-Checked Lattice Gauge Foundations, Certified Obstructions, and the Open Frontier

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22 November 2026

Abstract

We present an archival technical synthesis and handoff specification for the Yang–Mills formalization programme inside `THE-ERIKSSON-PROGRAMME` (archival release `yang-mills-archive-v1.0`). The repository pins Lean 4 (`v4.29.0-rc6`) and `mathlib` (`07642720480157414db592fa85b626dafb71355b`) and maintains a continuously verified core module, `YangMillsCore`, whose build and axiom oracle pass in GitHub Actions with zero `sorry` declarations, zero project-specific axioms, and every headline declaration restricted to Lean's three foundational axioms `[propext, Classical.choice, Quot.sound]`.

Within this verified core, the programme establishes: (i) a reusable Kotecký–Preiss and Mayer–Ursell polymer cluster-expansion engine (including Penrose's tree-graph inequality and zero-free polydiscs); (ii) finite-volume and volume-uniform strong-coupling Wilson-loop area laws and unconditional exponential decay of truncated plaquette correlations for SU(N) lattice gauge theory without invoking Peter–Weyl, together with fixed-spacing thermodynamic limits; (iii) exact $\mathbb{Z}_N$ centre-symmetry selection rules and fundamental Haar/Schur identities; (iv) machine-checked modified-Bessel/Amos ratio bounds and global ratio monotonicity for killed von Mises bridges; (v) complete Osterwalder–Seiler reflection-positivity and transfer-operator chains for $\mathbb{Z}_2$ lattice gauge systems; and (vi) certified negative obstructions isolating exact walls in block-Poincaré coercivity, flat lattice Hodge rank, spatial coupling, and sine-series monotonicity.

We state the limits with equal precision. No four-dimensional continuum limit ($a \to 0$), no physical SU(N) Osterwalder–Schrader or Wightman reconstruction, and no solution to the Clay Millennium Yang–Mills Mass Gap problem is claimed; the repository's historical Clay-distance shorthand remains $\sim$0% ($< 0.1\%$). Three open fronts are isolated and documented for future researchers and automated provers: the concrete Balaban/Dimock physical activity estimate `hRpoly` (frozen in draft PR #29 at 20/41 terminal producers and TermSource = 0), physical SU(N) reflection positivity and transfer-operator gaps beyond finite-cylinder geometry (R1/R2), and continuum/OS–Wightman reconstruction. All accompanying preprints in the AIRR archive (https://airr.science/) and reproduction steps are indexed in one citable record.

1 Scope, Verification Standard, and Epistemic Boundary

Formalizing constructive quantum field theory in an interactive theorem prover exposes two distinct failure modes: silent logical gaps (`sorry` or unproved `axiom` declarations) and vacuous formalizations whose top-level statements hold for degenerate models (such as trivial gauge groups

or coordinated dummy scalings). Early phases of `THE-ERIKSSON-PROGRAMME` encountered both; the mature repository enforces a strict archival discipline verified automatically on every push and pull request to `main`:

1. **Pinned toolchain and dependencies.** `lean-toolchain` pins `leanprover/lean4:v4.29.0-rc6`; `lakefile.lean` and `lake-manifest.json` pin `mathlib` to commit `07642720480157414db592fa85b626dafb71355b` and fix all transitive dependency revisions.

2. **Automated compilation and axiom oracle.** GitHub Actions workflow `lean-build.yml` builds `YangMillsCore` (recorded core count: 8466 Lake jobs in `project-state.json`) and runs `oracle_check.lean`, requiring every `#print axioms` readout to depend on nothing beyond `[propext, Classical.choice, Quot.sound]`.

3. **Zero-sorry and zero-core-axiom textual guard.** Workflow `ci.yml` runs `scripts/check_consistency.py`, a comment-aware scanner forbidding `sorry` across the proof tree and forbidding `axiom` declarations across the verified-core source directories (`L0_Lattice`, `L1_GibbsMeasure`, `KP`, `ClayCore`, `Paper`, `RG`, and the core `P8_PhysicalGap` Haar/compactness modules). Missing analytic theorems are carried strictly as explicit theorem hypotheses (`HYPOTHESIS_FRONTIER.md`), never as axioms.

4. **Explicit Clay-distance boundary.** The programme does *not* prove existence of a four-dimensional continuum Yang–Mills theory on $\mathbb{R}^4$, Osterwalder–Schrader or Wightman axioms for SU(N) ($N \geq 2$), or a continuum mass gap. The assembly lemmas in `YangMills/Paper/ClusteringToGap.lean` are explicitly documented as real-sequence exponential-decay lemmas; their physical role is conditional on genuine gauge-theoretic IR and UV inputs.

2 Summary of Machine-Checked Results in `YangMillsCore`

Table 1 summarizes the verified mathematical engines available for direct reuse by downstream formalization efforts and automated provers.

Layer	Verified Content (0 sorry, 0 project axioms)	Lean Location
KP / Mayer polymer engine	Mayer–Ursell inversion ($\Xi = \exp(\text{clusterSum})$), Penrose tree-graph inequality via greedy BFS partition schemes, polymer reconstruction $Z = \Xi$, Kotecký–Preiss convergence, and activity-domain zero-free polydiscs.	`YangMills/KP/`
Wilson area laws & IR clustering	Finite-volume and volume-uniform strong-coupling Wilson-loop area laws (linearized and exact activity); unconditional exponential decay of truncated plaquette correlations for SU(N) in an explicit non-empty coupling window without Peter–Weyl; fixed-spacing thermodynamic limit on integer coordinates.	`YangMills/L1_GibbsMeasure/` `paper/area-law/`
Haar & $\mathbb{Z}_N$ centre symmetry	Left-invariant vanishing engine, interacting Wilson-loop N-ality selection rules at any coupling β, fundamental Schur orthogonality on SU(N), U(1) character orthogonality on `AddCircle`, and certification of SU(1) degeneracy.	`YangMills/ClayCore/`
Conditional RG substrates	Block-spin averaging with gauge covariance and ℓ^2 contraction, finite-dimensional Gaussian pushforwards, exponential-decay kernel calculus, marginal-coupling 4D summability (`MarginalUVMassGap.lean`), and source-plus-defect error-budget landing pads (`YMActivityBudget`).	`YangMills/RG/` `YangMills/Paper/`
Bessel / Amos & 2D surface suite	Machine-checked Amos-type bounds for modified Bessel ratios at integer and real order, quantitative crossing classification, Turán-type monotonicity, sine-series Wronskian identity, and global ratio monotonicity for killed von Mises bridges.	`AmosClosure/` `MarkedRootedClosure/`
$\mathbb{Z}_2$ / $\mathbb{Z}_N$ OS & transfer chains	Complete machine-checked Osterwalder–Seiler chain for the $\mathbb{Z}_2$ gauge chain, degenerate reflection pairing and GNS quotient, Perron theorem for strictly positive kernels, spatial coupling obstructions, and congruence-spectrum rigidity.	`YangMills/OS/`
Finite SU(2) cylinder (R1/R2)	Four-vertex, six-link, two-plaquette SU(2) cylinder geometry, normalized product-Haar Gibbs measure, and exact reflection/Fubini identities (standalone, outside `YangMillsCore`; positivity and transfer remain open).	`docs/evidence/` `physical-cylinder-r2-20260906/`

Table 1: Verified mathematical engines in the archival release `yang-mills-archive-v1.0`.

3 Certified Obstructions and Negative Results

A central contribution of the programme for future researchers is a body of machine-checked negative theorems ruling out tempting but mathematically false shortcuts:

- **Volume-uniform Poincaré walls** (`ARR-2026-3Q0VH7PMNV8C9VQE`): obstructs flat and fluctuation-sector block-Poincaré routes to Combes–Thomas coercivity in lattice Yang–Mills.
- **Flat lattice Hodge factorized-penalty and coarse-rank walls** (`ARR-2026-627PF1B4KK9VTRYT`, `ARR-2026-7EPM3V3DP796BBRR`, and expanded revision dossier `papers/flat-coarse-rank/`): exact-kernel Fourier families obstruct every penalty factored through a block map in a flat lattice gauge form.
- **Spatial coupling and Birkhoff obstructions in $\mathbb{Z}_2$ slices** (`ARR-2026-4X084GVAYF87PB3F`, `ARR-2026-6M8PMVHTMH8YZ9ND`, `ARR-2026-1A608PTGTC9XX9JZ`): isolates where elementary uniform-vacuum and extent-independent spectral arguments break under spatial coupling.
- **Parity barriers for decoupling inequalities** (`ARR-2026-5HRC3PS9HZ9WDAA5`): no comparison functional of bounded marginal order can certify uniform decoupling.
- **Failure of Biernacki–Krzyż for sine series** (`papers/wronskian-reduction/`): explicit counterexample $a = (3, 4, 3)$, $b = (3, 2, 1)$ showing coefficient-ratio monotonicity cannot certify the sign of the antisymmetrized sine-sum Wronskian.
- **Two-transporter Haar projection design no-go** (`docs/SU2-TWO-TRANSPORTER-NOGO-20260731.md`): two free Haar-integrated transporters absent from the observable project the reduced SU(2) quadratic form to the trivial sector.

4 Open Frontiers for Continuation

Future human teams or autonomous theorem provers picking up this archive should start from `HORIZON.md`, `HYPOTHESIS_FRONTIER.md`, and `YangMills/CLOSURE.md`. Three concrete frontiers remain open:

1. **Physical UV activity estimate (`hRpoly`), frozen in draft PR #29.** Construct the concrete Balaban/Dimock cluster-expansion-with-holes remainder decay bound for the actual gauge RG operator. At the frozen September 2026 checkpoint (`docs/HRPOLY-STATUS.md`), the checklist records 20/41 terminal `PreEq136` producers, TermSource = 0, physical window 15 compatible but not attained, and uniform physical (B_0, δ_0) plus regional transport (F5) open.
2. **Physical SU(N) reflection positivity and transfer-operator spectral gap at fixed spacing.** Extend the verified R1/R2 finite SU(2) cylinder geometry and product-Haar measure to genuine complex Gram-matrix positivity retaining all crossing plaquette weights, a strictly positive centered norm, and a physical transfer operator (`docs/POST-HRPOLY-CONTINUUM-AUDIT-20260906.md`).
3. **Four-dimensional continuum limit ($a \to 0$) and Osterwalder–Schrader / Wightman reconstruction (M4–M5).** This remains open mathematics well beyond current

formalization infrastructure: removing the ultraviolet regulator while retaining a non-perturbative positive mass gap for SU(N) ($N \geq 2$) and constructing the continuum Hilbert space and fields.

Late-2026 open research branches on GitHub (not merged into `YangMillsCore`) preserve additional work on concentric integer-shell energy control and flat cochains (PRs #180–#190) and Boolean Doob chain retention criteria (PRs #292, #299).

5 Cross-Referenced AIRR Preprint Index

Table 2 links the core mathematical-physics manuscripts in `THE-ERIKSSON-PROGRAMME` to their stable records in the AIRR archive (https://airr.science/). Unrelated empirical audit manuscripts added under `papers/` in November 2026 are excluded from this index.

Repository Folder	AIRR Identifier	Topic / Role
paper/area-law	ARR-2026-2ZQ6F57N2H8MG91V	Volume-uniform Wilson-loop area law
papers/c1-rooted-tree-majorants	ARR-2026-505NQZHR7P9TQ8SA	Rooted-tree majorants for polymers with holes
papers/c2-amos-closure	ARR-2026-6DP8SDRZ678H9AY9	Amos Bessel-ratio calculus across three sites
papers/c3-amos-proof	ARR-2026-17K7KRR6NH9ZVAEJ	Machine-checked Amos bound (integer order)
papers/c4-amos-real	ARR-2026-3AJ7W5BS3N9VR8KM	Machine-checked Amos-type bound (real order)
papers/c5-crossing	ARR-2026-39Y0F83F4Z9JK90G	Diagonal Amos-type crossing classification
papers/c6-rg-interface	ARR-2026-0WG9YDY5BA9S3RF4	Non-circular RG interface for Wilson correlators
papers/bessel-amos-fh	ARR-2026-5PXDYNR2ER89HTQG	Order-monotonicity of $(\log I_\nu)'$ & 2D LGT
papers/phi-lemma	ARR-2026-72G1SSCNNG8YDVC2	Weighted Turán-type Bessel monotonicity
papers/surface-theorem	ARR-2026-1FPFA1TSMY9M3T9S	Killed von Mises bridge ratio monotonicity
papers/surface-complete	ARR-2026-3CEWH763KG84B8TM	Global ratio monotonicity (closure)
papers/poincare-wall	ARR-2026-3Q0VH7PMNV8C9VQE	Volume-uniform Poincaré walls
papers/flat-coarse-rank	ARR-2026-7EPM3V3DP796BBRR	Flat Hodge factorized-penalty wall (v2 dossier)
papers/parity-barriers	ARR-2026-5HRC3PS9HZ9WDAA5	Parity barriers for decoupling inequalities
papers/o-bridge	ARR-2026-31ED4MTKC18JPSDE	Dense-family clustering-to-gap criterion
papers/reflection-positivity	ARR-2026-10JJK0VCCS87VTQH	$\mathbb{Z}_N$ reflection positivity ($\mathbb{Z}_2$ instance)
papers/os-quotient	ARR-2026-2TBRDN2DE598KSXD	Degenerate reflection pairing & GNS quotient
papers/perron-kernel	ARR-2026-2YDE42KFDY8Y3AA0	Perron theorem for strictly positive kernels
papers/perron-gap	ARR-2026-1A608PTGTC9XX9JZ	Finite-extent spectral gap (not uniform)
papers/spatial-extent	ARR-2026-4X084GVAYF87PB3F	Spatial coupling breaks uniform vacuum
papers/spatial-birkhoff	ARR-2026-6M8PMVHTMH8YZ9ND	Second spatial-extent obstruction
papers/spatial-gibbs	ARR-2026-2TTA0WSQBT89X964	Transfer bridge for spatial $\mathbb{Z}_2$ slice
papers/spatial-reflection	ARR-2026-02YRWX53DG9M3R0R	Endpoint reflection positivity ($\mathbb{Z}_2$ slice)
papers/spatial-os	ARR-2026-223XBWQ8PQ8Z2RDS	Bond reflection positivity & site form
papers/spatial-spectral	ARR-2026-1A7WS4SDF0894BPS	Fluctuation-sector operator bound

Repository Folder	AIRR Identifier	Topic / Role
papers/spatial-uniform	ARR-2026-3EXXMXJD119J1R6R	Extent-independent rate (decoupled slice)
papers/congruence-spectrum	ARR-2026-7S1XA3PZEB9QM8FQ	Congruence rigidity and the fusion bound
papers/su2-crossing-compensated	ARR-2026-5V5K6T6DQT9J8SY6	Compensated-flow SU(2) crossing Ward
papers/thermodynamic-limit-kp	ARR-2026-5SGNYDASRB9V9VF9	Thermodynamic limit for local Gibbs states
papers/stationary-doob347	ARR-2026-1AS15VP0199VZ847	Boolean Doob kernels: stationary marginals
papers/stationary-weighting354	ARR-2026-4T35VQ6MZB8F99F4	Boolean Doob kernels: moment transfer
papers/os-chain-z2	*Pending AIRR deposit*	Complete $\mathbb{Z}_2$ Osterwalder–Seiler chain
papers/wronskian-reduction	*Pending AIRR deposit*	Wronskian identity for antisymmetrized sine sums

Table 2: Cross-reference of Yang–Mills and mathematical-physics manuscripts in THE-ERIKSSON-PROGRAMME to AIRR archive identifiers.

6 Reproducibility and Citation

To reproduce the verified Lean 4 core from scratch (on Windows, pass `-c core.longpaths=true` due to nested historical audit paths under `papers/marginal-transfer/`):

```
git clone -c core.longpaths=true \
  https://github.com/lluiseriksson/THE-ERIKSSON-PROGRAMME.git
cd THE-ERIKSSON-PROGRAMME
git checkout yang-mills-archive-v1.0
lake exe cache get
lake build YangMillsCore
lake env lean oracle_check.lean
python scripts/check_consistency.py
```

The code is released under AGPL-3.0-or-later with machine-readable citation metadata in CITATION.cff and .zenodo.json.

Disclosure of AI Assistance and Editorial Independence

Parts of the Lean 4 formalization, audit tooling, and manuscript preparation in THE-ERIKSSON-PROGRAMME were assisted by large language models under the supervision of the author; every core mathematical claim is independently checked by the Lean 4 kernel (YangMillsCore) with standard axioms only. The author (Lluis Eriksson) administers the AIRR archive (https://airr.science/); accordingly, this technical note claims no independent editorial admission or peer-review endorsement beyond its transparent machine-checked evidence and explicit scope boundaries.