The Eriksson Programme (Yang–Mills): Auditable Claim-to-Theorem Manifest, Certified Obstructions, and Continuation Frontier (Release `yang-mills-archive-v1.0`)

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23 November 2026 (v3)

Abstract

This archival technical note provides a self-contained, auditable claim-to-theorem specification and continuation handoff for the Yang–Mills Lean 4 formalization inside `THE-ERIKSSON-PROGRAMME` at archival release `yang-mills-archive-v1.0` (commit `6b54eca695e96b3038ceda18276c4e28fb45983f`; canonical Lean core checkpoint `02fae0ad` in `project-state.json`, Lean `v4.29.0-rc6`, pinned `mathlib` `07642720480157414db592fa85b626dafb71355b`). Responding to every material objection raised by hostile reviews of v1 and v2, this v3 revision supplies exact fully-qualified Lean 4 theorem statements and complete hypotheses (including the complex modulus and all four KP inequalities in the exact-action Wilson-loop area law, the unspannable-chain behavior of `chainAreaA`, the $\beta > 0$ hypothesis and literal definitions of `SingleScaleUVDecay` and `covUV_concrete` in the marginal RG conditional, the unweighted Poincaré predicate vs. penalized coercivity and exact scale quantifiers in the Poincaré walls, the explicit transverse periodic Fourier witness $A^{(\ell)} \in \ker Q_\ell$ for the flat Hodge factorized-penalty wall, the explicit `AmosBound` inequality, and the `CompatibleLocalObservable` thermodynamic-limit substrate), a declaration-level audit-coverage table distinguishing root `oracle_check.lean` (2,851 readouts across 46 imports), per-module oracles, and compiled-only modules, and a reconciled cross-reference of all 33 companion mathematical-physics repository folders to their 33 live AIRR identifiers.

The verified core proves fixed-spacing lattice results only: a Kotecký–Preiss / Mayer–Ursell polymer engine; volume-uniform strong-coupling SU(N_c) Wilson-loop area laws and exponential clustering of truncated two-plaquette correlations without Peter–Weyl; explicit strictly-interacting SU(2) thermodynamic-limit witnesses in $d = 2$ ($0 < |\beta| \leq 10^{-5}$) and $d = 4$ ($0 < |\beta| \leq 10^{-6}$); exact $\mathbb{Z}_n$ centre-symmetry selection rules; conditional real-sequence 4D marginal-coupling UV-summation theorems; real-order Amos/Bessel ratio bounds; and a transfer-matrix identification chain for the 1D open $\mathbb{Z}_2$ Ising spin chain. No four-dimensional continuum limit ($a \to 0$), no physical SU(N) Osterwalder–Schrader or Wightman reconstruction, and no solution to the Clay Millennium Yang–Mills Mass Gap problem is claimed.

1 Verification Architecture and Declaration-Level Coverage

Release `yang-mills-archive-v1.0` maintains four distinct automated layers, which must not be conflated:

1. **Root axiom oracle (`oracle_check.lean`, checked by `lean-build.yml`):** imports `YangMillsCore` plus 45 explicit `YangMills.{OS,RG,Paper,SUSY}.*` modules (46 imports total) and executes **2,851** `#print axioms` readouts. `scripts/check_oracle_output.py` parses the Lean

stdout and fails on any `sorryAx`, non-standard axiom outside `[propext, Classical.choice, Quot.sound]`, warning, or missing/reordered readout.

2. **Per-module axiom oracles:** separate oracle files audit specialized tracks, notably `AmosClosure/Oracle.lean` (real/integer Amos suite) and `YangMills/RG/UniformPhysicalCoarseRankWallOracle.lean` (plus sister `YangMills/RG/*Oracle.lean` files for the flat Hodge/coarse-rank obstructions).

3. **Core-compiled declarations outside the root oracle:** modules compiled by `lake build YangMillsCore` (e.g. `ThermodynamicLimit.lean`, `ThermodynamicNonvacuity.lean`, `PhysicalPoincareWall.lean`) are kernel-checked at build time and scanned by `scripts/check_consistency.py` for `sorry` and `axiom`, but are not listed in the root `oracle_check.lean`.

4. **Standalone / branch-only modules (outside `YangMillsCore`):** the finite SU(2) cylinder modules (R1/R2, Sept 2026) and draft PR #29 (`hRpoly`) are excluded from `YangMillsCore.lean`.

Table 1 maps each headline claim in Sections 2–3 to its exact Lean declaration and verification tier.

Headline Claim	**Fully-Qualified Lean Declaration**	**Audit Tier**
KP / Mayer inversion	`YangMills.KP.partition_eq_exp_clusterSum_of_kp`	Root `oracle_check.lean`
Volume-uniform area law	`YangMills.normalized_exp_wilson_loop_area_law`	Root `oracle_check.lean`
Exp. area-law decay	`YangMills.area_law_to_exp_area_decay`	Root `oracle_check.lean`
SU(N_c) 2-plaquette decay	`YangMills.sun_two_plaquette_correlator_bound`	Root `oracle_check.lean`
Fixed-lattice clustering	`YangMills.sun_lattice_exponential_clustering`	Root `oracle_check.lean`
$\mathbb{Z}_n$ centre selection	`YangMills.integral_wilsonLoopSU_listProd_gibbs_eq_zero`	Root `oracle_check.lean`
Marginal RG conditional	`YangMills.RG.lattice_mass_gap_of_singleScaleUVDecay_marginal`	Root `oracle_check.lean`
1D $\mathbb{Z}_2$ transfer chain	`YangMills.OS.z2_identification`, `z2_two_point_tanh`, `z2TransferOp_gap`, `z2_mass_pos`	Root `oracle_check.lean`
Thermodynamic limit	`YangMills.WindowPolymer.infiniteLocalGibbsState`	Compiled (`YangMillsCore`)
SU(2) $d = 2, 4$ witnesses	`YangMills.su2D4InfiniteLocalGibbsStateOnPuncturedInterval`	Compiled (`YangMillsCore`)
Real-order Amos suite	`AmosClosure.amosBoundReal_holds`, `besselIReal_unit_step`	`AmosClosure/Oracle.lean`
Poincaré walls	`YangMills/RG/PhysicalPoincareWall.lean`, `PhysicalPoincareLowModeFalsifier.lean`	Compiled (`YangMillsCore`)
Flat coarse-rank wall	`YangMills/RG/UniformPhysicalCoarseRankWall.lean`	`UniformPhysicalCoarseRankWallOracle.lean`

Table 1: Declaration-level mapping between headline claims and automated audit coverage at commit `6b54eca695e96b3038ceda18276c4e28fb45983f`.

2 Auditable Claim-to-Theorem Manifest

Every statement below quotes the exact mathematical content and hypotheses of the corresponding Lean 4 declaration at commit `6b54eca695e96b3038ceda18276c4e28fb45983f`. No prior-work pri-

ority claim is made for the classical lattice theorems; the contribution is Lean 4 mechanization and the certified obstruction map.

2.1 KP / Mayer polymer engine (`YangMills/KP/`)

- `YangMills.KP.partition_eq_exp_clusterSum_of_kp` (`KP/MayerInversion.lean`): for any finite polymer model P (`[Fintype P.Polymer]`) and weight $a : P.\text{Polymer} \to \mathbb{R}$ satisfying `KPCriterion P a`, the polymer partition function equals $\Xi = \exp(\text{clusterSum } P)$ with absolutely summable cluster series, backed by Penrose's tree-graph bound (`abs_ursell_le_card_spanningTrees`).

2.2 Strong-coupling $\mathrm{SU}(N_c)$ area law, IR clustering, thermodynamic limit, and centre selection

- **Volume-uniform exact-action Wilson-loop area law** (`YangMills.normalized_exp_wilson_loop_area_law`, `L1_GibbsMeasure/RestrictedGate.lean`): let $d, N, N_c \geq 1$ (`[NeZero d] [NeZero N] [NeZero N_c]`), let es : List (ConcreteEdge d N) be an arbitrary edge list, let $\delta \geq 0$, c, c' : ConcretePlaquette d $N \to \mathbb{C}$ satisfy $\|c_p\| \leq \delta$, $\|c'_p\| \leq \delta$, and $c'_p = \mathrm{conj}(c_p)$, and let $t, \varepsilon \geq 0$, $\sigma \in [0,1]$. Define the activity bound $u := e^{2\delta N_c} - 1$ and require all four volume-independent inequalities:

$$(16d+1)^2 u\, e^{t+\varepsilon+1} < 1, \qquad 16d\, \frac{u\, e^{t+\varepsilon+1}}{1-(16d+1)^2 u\, e^{t+\varepsilon+1}} \leq t,$$
$$(16d+1)^2 \sigma < 1, \qquad u\, \exp\!\left(16d\, e\, \frac{u\, e^{t}}{1-(16d+1)^2 u\, e^{t}}\right) \leq \sigma^2.$$

  Set $K_1 := e\,\frac{ue^t}{1-(16d+1)^2 ue^t}$ and $K_2 := \frac{\sigma}{1-(16d+1)^2\sigma}$, and let $Z := \texttt{weightedPartition}(\mu_{\mathrm{SU}(N_c)}, \texttt{expReActivity}\ N_c\ c) > 0$. Then the complex modulus of the normalized expectation under the exact plaquette action $W_c(A) := \prod_p \exp\!\big(c_p \,\mathrm{tr}(U_p) + c'_p\ \mathrm{conj}(\mathrm{tr}(U_p))\big)$ satisfies

$$\left\| \frac{1}{Z} \int \mathrm{tr}(U_{es})\, W_c(A)\, d\mu_{\mathrm{Haar}} \right\| \;\leq\; N_c\, e^{4d\,|\mathrm{edgeSupport}(es)|(K_1+K_2)}\, \sigma^{\,\mathrm{chainAreaA}_{\mathbb{Z}/N_c\mathbb{Z}}(\mathrm{loopChain}(es))}.$$

  Here `chainAreaA` (`L0_Lattice/ChainComplex.lean`) is defined as $\mathrm{Nat.sInf}\{n \in \mathbb{N} \mid \exists s : P \to \mathbb{Z}/N_c\mathbb{Z},\ \partial_2^{\mathrm{anti}} s = c \wedge |\mathrm{supp}(s)| = n\}$. For unspannable 1-chains (such as open edge words or homologically nontrivial torus cycles), the set is empty and Lean's `Nat.sInf` returns the junk value 0, so $\sigma^0 = 1$ and the inequality holds trivially with no area suppression. Likewise, a twice-traversed SU(2) loop has $\mathrm{loopChain}_{\mathbb{Z}/2\mathbb{Z}} = 0$, hence chainAreaA $= 0$ ($\sigma^0 = 1$), consistent with the free ($\beta = 0$) Haar value $\mathbb{E}[\mathrm{tr}(U_C^2)/2] = -1/2$ for any simple (non-backtracking) loop C whose holonomy U_C is Haar-distributed on SU(2).

- **Unconditional $\mathrm{SU}(N_c)$ two-plaquette exponential clustering without Peter–Weyl** (`YangMills.two_plaquette_correlator_bound` and its $\mathrm{SU}(N_c)$ specialization `YangMills.sun_two_plaquette_correlator_bound`, `L1_GibbsMeasure/TwoPlaquetteCorrelator.lean`): for any measurable $f : \mathrm{SU}(N_c) \to \mathbb{R}$ with $|f| \leq 1$, any $s > 0$, $\beta \in \mathbb{R}$, $t, \varepsilon \geq 0$, setting $v := (e^{|\beta| N_c} - 1) + s + (e^{|\beta| N_c} - 1)s$ and $Q := \frac{v\, e^{t+\varepsilon+1}}{1-(16d+1)^2 v\, e^{t+\varepsilon+1}}$, assume $(16d+1)^2 v\, e^{t+\varepsilon+1} < 1$, $16d\,Q \leq t$, and the paired distance condition $4e^{-\varepsilon k} Q \leq 1$ for distinct plaquettes $p \neq q$ with

$2k \le \mathrm{dist}_{\mathrm{touchGraph}}(p,q)$. Then the unnormalized connected correlator satisfies

$$\left|\langle f(U_p)f(U_q)\rangle_{\mathrm{unnorm}} Z - \langle f(U_p)\rangle_{\mathrm{unnorm}}\langle f(U_q)\rangle_{\mathrm{unnorm}}\right| \;\le\; \frac{8Q(1+s)^2Z^2}{s^2}\, e^{-\varepsilon k}.$$

(The theorem allows $\varepsilon \ge 0$; nontrivial exponential decay requires choosing $\varepsilon > 0$, as supplied by the concrete witnesses below.)

- **Fixed-spacing thermodynamic limit and strictly-interacting** SU(2) **witnesses** (`LocalObservableSubstrate.lean`, `ThermodynamicLimit.lean`, `ThermodynamicNonvacuity.lean`): observables belong to `CompatibleLocalObservable d G` — a finite edge support `Support` embedded injectively into $(\mathbb{N}^d \times \mathrm{Fin}\, d)$ inside a box of side minVolume + 1, equipped with a measurable bounded kernel $K : (\mathrm{Support} \to G) \to \mathbb{R}$ ($|K| \le B$) realized on periodic tori of side $n + 1 \ge \mathrm{minVolume} + 1$ via modulo coordinate reduction `edgeAt`. Under `UniformLocalKPRegime d B beta` (with $\varepsilon > 0, \eta > 0$), finite-torus expectations form a Cauchy sequence in $\mathbb{R}$ with exponential tail control as $n \to \infty$, converging pointwise on `CompatibleLocalObservable d G` to a linear, positive, normalized, translation-invariant functional `PositiveNormalizedLocalState d G` (`YangMills.WindowPolymer.infiniteLocalGibbsState`), and transferring two-plaquette clustering to the infinite-volume state. `ThermodynamicNonvacuity.lean` constructs two concrete strictly-interacting SU(2) families (nonconstant plaquette energy $\mathrm{Re}\,\mathrm{tr}\, U$, $B = 2$, $t = \varepsilon = \eta = 10^{-2} > 0$, explicitly requiring $0 < |\beta|$):
  - $d = 2$, $0 < |\beta| \le 10^{-5}$: `su2InfiniteLocalGibbsStateOnPuncturedInterval`;
  - $d = 4$, $0 < |\beta| \le 10^{-6}$: `su2D4InfiniteLocalGibbsStateOnPuncturedInterval`.

  Both instantiate `PositiveNormalizedLocalState d SU2GaugeGroup` with strictly positive decay parameter $\varepsilon = 10^{-2}$ and nonzero coupling.

- **Exact $\mathbb{Z}_n$ centre-symmetry selection rules** (`L1_GibbsMeasure/GibbsSelectionRule.lean`, e.g. `integral_wilsonLoopSU_gibbs_eq_zero` and `integral_wilsonLoopSU_listProd_gibbs_eq_zero`): multiplying every positively oriented edge by $\omega = e^{2\pi i/n} I_n \in Z(\mathrm{SU}(n))$ leaves every plaquette invariant for all $\beta \in \mathbb{R}$ (two forward and two backward edges), while any positively-oriented edge list es acquires factor $\omega^{|es|}$, forcing vanishing interacting Gibbs expectation whenever $n \nmid |es|$.

2.3 Conditional 4D marginal-coupling RG assembly (`YangMills/RG/`)

- `YangMills.RG.lattice_mass_gap_of_cluster_and_marginal_coupling` and `lattice_mass_gap_of_singleScaleUVDecay_marginal` (`RG/MarginalUVMassGap.lean:111-148`): given real sequences $\mathrm{covIR}, g : \mathbb{N} \to \mathbb{R}$, $R_{\mathrm{sc}} : \mathbb{N} \to \mathbb{N} \to \mathbb{R}$, $n_{\mathrm{sc}} : \mathbb{N} \to \mathbb{N}$, define

$$\texttt{covUV_concrete}\; R_{\mathrm{sc}}\; n_{\mathrm{sc}}\; t := \sum_{k=0}^{n_{\mathrm{sc}}(t)-1} R_{\mathrm{sc}}(t,k),$$

  and the predicate

$$\texttt{SingleScaleUVDecay}\; R_{\mathrm{sc}}\; g\; C_2\; c_0\; \kappa_0 \;:\Longleftrightarrow\; \forall t,k \in \mathbb{N},\; |R_{\mathrm{sc}}(t,k)| \le \left(C_2\, e^{-c_0 t}\right) g_k^{\kappa_0}.$$

  Under the complete hypotheses $0 < \varepsilon$, $0 < c_0$, $0 \le C_2$, $1 < \kappa_0$, $0 < \beta$, $\forall k,\ 0 < g_k$, $\forall k,\ \beta g_k < 1$, $\forall k,\ g_{k+1} = g_k(1 - \beta g_k)$, $\forall k,\ |\mathrm{covIR}_k| \le C_1 e^{-\varepsilon k}$, and `SingleScaleUVDecay Rsc g C2 c0 kappa0`, the theorem derives $1/g_k \ge 1/g_0 + \beta k$ (hence $g_k = O(1/k)$, so $\sum_{k=0}^{\infty} g_k^{\kappa_0} < \infty$ whenever

$\kappa_0 > 1$ by p-series comparison, whereas at $\kappa_0 = 1$ this $O(1/k)$ upper bound is not summable) and concludes:

$$\exists\, \mathrm{gap} > 0,\ \forall t \in \mathbb{N}, \quad \left|\mathrm{covIR}_t + \texttt{covUV_concrete}\ R_{\mathrm{sc}}\ n_{\mathrm{sc}}\ t\right| \le \Big(C_1 + C_2 \sum_{k=0}^{\infty} g_k^{\kappa_0}\Big) e^{-\mathrm{gap}\cdot t}.$$

This is an abstract real-sequence assembly mentioning no gauge group or Hilbert space; discharging the physical single-scale input `hRpoly` remains open.

2.4 Real-order Amos/Bessel suite and 1D $\mathbb{Z}_2$ transfer chain

- **Real-order Amos bounds (`AmosClosure/Core.lean`, `AmosBarrierReal.lean`):** defining $\psi_\nu(x) = x(r_\nu(x)^{-1} - r_\nu(x))$ for $r_\nu(x) = I_{\nu+1}(x)/I_\nu(x)$ and

$$\texttt{AmosBound}\ \nu\ x\ r \ :\Longleftrightarrow\ r < \frac{x}{\nu + \frac{1}{2} + \sqrt{(\nu+\frac{1}{2})^2 + x^2}},$$

the Lean suite proves $2\nu + 1 < \psi_\nu(x)$ (`besselPsiReal_gt`), `AmosBound` for $r_\nu(x)$ (`amosBoundReal_holds`), and the unit-step bound $r_\nu(x) - r_{\nu+1}(x) < 1/x$ (`besselIReal_unit_step`) for all real $\nu \ge 0$ and all $x > 0$ (audited in `AmosClosure/Oracle.lean`).

- **1D open $\mathbb{Z}_2$ Ising transfer-matrix chain (`ARR-2026-7PRDH0DHH280HRKH` v3):** machine-checks $Z_n = 2(e^\beta + e^{-\beta})^n$, $\mathbb{E}_n[\mathrm{conj}(A(\sigma_0))B(\sigma_n)] = \langle A\Omega, T_\beta^n(B\Omega)\rangle_{\mathbb{C}}$, $\mathbb{E}_n[f(\sigma_0)f(\sigma_n)] = (\tanh\beta)^n$, $\|T_\beta - P\| \le \tanh\beta$ ($\beta \ge 0$), and time-zero pairing non-degeneracy. Analytically, $1 - |\tanh\beta|$ is the *absolute* spectral gap across all β; for $\beta < 0$ the non-vacuum eigenvalue $\tanh\beta$ is negative, so T_β has no self-adjoint real logarithm H with $e^{-H} = T_\beta$. Positive kernel entries alone do not imply reflection positivity, and finite-extent Perron isolation does not imply a uniform gap.

3 Certified Negative Results and Exact Obstruction Classes

1. **Volume-uniform Poincaré walls (`ARR-2026-3Q0VH7PMNV8C9VQE`):** on the unit-spacing periodic torus $\Lambda_{LN'} = (\mathbb{Z}/LN'\mathbb{Z})^d$ ($d \ge 3$, $N_c \ge 2$, internal space $\mathbb{R}^{N_c^2-1}$ with concrete matricial $\mathfrak{su}(N_c)$ adjoint model), let $K_0 = D_1^\dagger D_1 + D_0 D_0^\dagger$ be the flat lattice Hodge operator ($\langle A, K_0 A\rangle = \|\mathrm{curl}\, A\|^2 + \|\mathrm{div}\, A\|^2$) and let Q be the *unscaled* line-integral block map satisfying $Q(A_v) = L\, A_v^{\mathrm{coarse}}$ on direction-wise constants. The formalized Poincaré predicate `FlatGaugeHodgePoincare d L N' Nc rho CP` is the *unweighted* inequality

$$0 < C_P \quad \wedge \quad \forall A, \quad \|A\|^2 \le C_P\, (\langle A, K_0 A\rangle + \|QA\|^2),$$

which feeds coercivity $c = \min(1, a)/C_P$ to the penalized covariance $K = K_0 + aQ^\dagger Q$ ($a > 0$). Two walls refute uniformity as block side grows:

   - *Constant-sector wall:* on side LN' for any fixed $N' \ge 1$, testing on A_v ($\langle A_v, K_0 A_v\rangle = 0$, $\|A_v\|^2 = (LN')^d\|v\|^2$, $\|QA_v\|^2 = L^2(N')^d\|v\|^2$) forces $C_P \ge L^d/L^2 = L^{d-2} \to \infty$ as $L \to \infty$ (this refutes uniformity in L, not in N' at fixed L).
   - *Fluctuation-sector wall:* restricting to cochains orthogonal to constants on even block side $L = 2M$ (fine side $(2M)N'$, any fixed $N' \ge 1$), a transported half-period square-wave mode satisfies $\|QA\|^2 \le (2M)^{-1}\|A\|^2$ and $\langle A, K_0 A\rangle = 8((2M)N')^{-1}\|A\|^2$. Because the Poincaré predicate carries unit weight on $\|QA\|^2$ and $N' \ge 1$, the Rayleigh numerator is at most $(\frac{8}{(2M)N'} + \frac{1}{2M})\|A\|^2 \le \frac{9}{2M}\|A\|^2$, forcing $C_P \ge 2M/9 \to \infty$ as $M \to \infty$ for every $N' \ge 1$, $d \ge 3$, $N_c \ge 2$, and every adjoint model.

2. **Flat lattice Hodge factorized-penalty and coarse-rank walls** (`ARR-2026-627PF1B4KK9VTRYT`, `ARR-2026-7EPM3V3DP796BBRR`, consolidated dossier `papers/flat-coarse-rank/`): on $X_n = (\mathbb{Z}/n\mathbb{Z})^d$ ($n = \ell N'$, $d \geq 2$, $N_c \geq 2$, unit-spacing forward-difference curl $\mathrm{d}A$ and backward divergence δA, $\langle A, K_{0,n} A\rangle = \|\mathrm{d}A\|^2 + \|\delta A\|^2$) with the averaged line-integral block map $(Q_\ell A)_i(y) = \ell^{-d} \sum_{x \in B_y} \sum_{k=0}^{\ell-1} A_i(x + ke_i)$, pick distinct directions $p \neq q$, orthogonal internal unit vectors $u, v \in \mathbb{R}^{N_c^2-1}$, $\iota(z) = (\mathrm{Re}\, z)u + (\mathrm{Im}\, z)v$, and define for $\ell \geq 2$ the explicit transverse ℓ-periodic Fourier witness:

$$A_i^{(\ell)}(x) := \mathbf{1}_{i=p}\, \iota\big(e^{2\pi i x_q/\ell}\big).$$

Because $\sum_{r_q=0}^{\ell-1} e^{2\pi i r_q/\ell} = 0$ and shifts along $p \neq q$ leave x_q unchanged, $Q_\ell A^{(\ell)} = 0$ identically; meanwhile $\delta A^{(\ell)} = 0$, $\|A^{(\ell)}\|^2 = (\ell N')^d$, and $\langle A^{(\ell)}, K_{0,n} A^{(\ell)}\rangle = (\ell N')^d \cdot 4\sin^2(\pi/\ell)$ (nonzero, decaying as $O(\ell^{-2})$ — not in $\ker K_{0,n}$). Consequently, for *every* penalty $\Psi_\ell(Q_\ell A)$ satisfying $\Psi_\ell(0) = 0$ (including any factorized quadratic form $Q_\ell^* M_\ell Q_\ell$ and nonlinear penalties), $C_P(\ell) \geq (4\sin^2(\pi/\ell))^{-1} \geq \ell^2/(4\pi^2) \to \infty$ as $\ell \to \infty$. Beyond fixed Q_ℓ, `UniformPhysicalCoarseRankWall.lean` constructs a low-frequency real subspace S_R of dimension $2(2R+1)^d$ with $\langle A, K_{0,n} A\rangle \leq (4\pi^2 d R^2/n^2)\|A\|^2$, ruling out uniform coercivity from any fixed-dimensional linear coarse measurement plus uniformly bounded-rank fine repair.

3. **Sine-series Wronskian counterexamples (`ARR-2026-5FKM2GGK348CPTE9` v3):** using the doubled Wronskian convention $W(t) := 2[F_a'(t)F_b(t) - F_a(t)F_b'(t)]$ from Theorem 1 of that paper, $a = (3,4,3), b = (3,2,1)$ gives $W(\pi/2) = -32 < 0$ and $W(2\pi/3) = 3\sqrt{3} > 0$, while $a = (6,10,24), b = (6,5,2)$ gives $W(\pi - \varepsilon) = -8\varepsilon^3 + O(\varepsilon^5) < 0$ yet $W(2\pi/3) = 51\sqrt{3} > 0$, refuting unrestricted sine-series Biernacki–Krzyż and generic local-to-global lifting.

4. **Additional programme obstructions (see Table 2 for complete statements):** spatial coupling breaks the uniform vacuum and finite-extent Perron isolation fails to give extent-uniform gaps (`ARR-2026-4X084GVAYF87PB3F`, `ARR-2026-6M8PMVHTMH8YZ9ND`, `ARR-2026-1A608PTGTC9XX9JZ`); bounded-order marginal functionals face parity barriers in decoupling inequalities (`ARR-2026-5HRC3PS9HZ9WDAA5`); and integrating out two free Haar transporters absent from an observable projects the reduced SU(2) form to the trivial sector (`docs/SU2-TWO-TRANSPORTER-NOGO-20260731.md`).

4 Open Formalization Tasks vs. Open Mathematics

1. **Open formalization task — fixed-spacing physical** SU(N) **Wilson reflection positivity:** classically proved by Osterwalder and Seiler (1978), but open in this Lean archive beyond standalone R1/R2 finite SU(2) cylinder modules outside `YangMillsCore`.

2. **Open formalization / constructive-QFT task — `hRpoly` (draft PR #29):** concrete Balaban/Dimock cluster-expansion-with-holes remainder decay for the physical gauge RG operator (frozen checkpoint: 20/41 terminal `PreEq136` producers, TermSource = 0, uniform physical (B_0, δ_0) and regional transport F5 open).

3. **Open mathematics — 4D continuum limit ($a \to 0$) and OS/Wightman reconstruction with mass gap:** removing the UV regulator for SU(N) ($N \geq 2$) while establishing a non-perturbative mass gap remains the Clay Millennium Problem.

5 Resolved AIRR Preprint Index (33 Companion Folders)

Table 2 maps all **33** companion mathematical-physics folders in `THE-ERIKSSON-PROGRAMME` to their 33 live AIRR identifiers on https://airr.science/ (together with this archival technical note `ARR-2026-7DYZAEKGJP8FPTJ1`, 34 total). Unrelated empirical audit manuscripts added under `papers/` in November 2026 are excluded.

Repository Folder	AIRR Identifier	Topic / Role
`paper/area-law`	`ARR-2026-2ZQ6F57N2H8MG91V`	Volume-uniform Wilson-loop area law
`papers/c1-rooted-tree-majorants`	`ARR-2026-505NQZHR7P9TQ8SA`	Rooted-tree majorants for polymers with holes
`papers/c2-amos-closure`	`ARR-2026-6DP8SDRZ678H9AY9`	Amos Bessel-ratio calculus across three sites
`papers/c3-amos-proof`	`ARR-2026-17K7KRR6NH9ZVAEJ`	Machine-checked Amos bound (integer order)
`papers/c4-amos-real`	`ARR-2026-3AJ7W5BS3N9VR8KM`	Machine-checked Amos-type bound (real order)
`papers/c5-crossing`	`ARR-2026-39Y0F83F4Z9JK90G`	Diagonal Amos-type crossing classification
`papers/c6-rg-interface`	`ARR-2026-0WG9YDY5BA9S3RF4`	Non-circular RG interface for Wilson correlators
`papers/bessel-amos-fh`	`ARR-2026-5PXDYNR2ER89HTQG`	Order-monotonicity of $(\log I_\nu)'$ & 2D LGT
`papers/phi-lemma`	`ARR-2026-72G1SSCNNG8YDVC2`	Weighted Turán-type Bessel monotonicity
`papers/surface-theorem`	`ARR-2026-1FPFA1TSMY9M3T9S`	Killed von Mises bridge ratio monotonicity
`papers/surface-complete`	`ARR-2026-3CEWH763KG84B8TM`	Global ratio monotonicity (closure)
`papers/wronskian-reduction`	`ARR-2026-5FKM2GGK348CPTE9`	Sine-series Wronskian, $F_B > 0$ proof & counterexamples
`papers/poincare-wall`	`ARR-2026-3Q0VH7PMNV8C9VQE`	Volume-uniform Poincaré walls
`papers/flat-coarse-rank`	`ARR-2026-7EPM3V3DP796BBRR`	Flat Hodge factorized-penalty wall (v2 dossier)
`papers/parity-barriers`	`ARR-2026-5HRC3PS9HZ9WDAA5`	Parity barriers for decoupling inequalities
`papers/o-bridge`	`ARR-2026-31ED4MTKC18JPSDE`	Dense-family clustering-to-gap criterion
`papers/reflection-positivity`	`ARR-2026-10JJK0VCCS87VTQH`	$\mathbb{Z}_N$ reflection positivity ($\mathbb{Z}_2$ instance)
`papers/os-chain-z2`	`ARR-2026-7PRDH0DHH280HRKH`	Open $\mathbb{Z}_2$ Ising transfer-matrix chain
`papers/os-quotient`	`ARR-2026-2TBRDN2DE598KSXD`	Degenerate reflection pairing & GNS quotient
`papers/perron-kernel`	`ARR-2026-2YDE42KFDY8Y3AA0`	Perron theorem for strictly positive kernels
`papers/perron-gap`	`ARR-2026-1A608PTGTC9XX9JZ`	Finite-extent spectral gap (not uniform)
`papers/spatial-extent`	`ARR-2026-4X084GVAYF87PB3F`	Spatial coupling breaks uniform vacuum
`papers/spatial-birkhoff`	`ARR-2026-6M8PMVHTMH8YZ9ND`	Second spatial-extent obstruction
`papers/spatial-gibbs`	`ARR-2026-2TTA0WSQBT89X964`	Transfer bridge for spatial $\mathbb{Z}_2$ slice
`papers/spatial-reflection`	`ARR-2026-02YRWX53DG9M3R0R`	Endpoint reflection positivity ($\mathbb{Z}_2$ slice)
`papers/spatial-os`	`ARR-2026-223XBWQ8PQ8Z2RDS`	Bond reflection positivity & site form
`papers/spatial-spectral`	`ARR-2026-1A7WS4SDF0894BPS`	Fluctuation-sector operator bound
`papers/spatial-uniform`	`ARR-2026-3EXXMXJD119J1R6R`	Extent-independent rate (decoupled slice)
`papers/congruence-spectrum`	`ARR-2026-7S1XA3PZEB9QM8FQ`	Congruence rigidity and the fusion bound
`papers/su2-crossing-compensated`	`ARR-2026-5V5K6T6DQT9J8SY6`	Compensated-flow SU(2) crossing Ward
`papers/thermodynamic-limit-kp`	`ARR-2026-5SGNYDASRB9V9VF9`	Thermodynamic limit for local Gibbs states
`papers/stationary-doob347`	`ARR-2026-1AS15VP0199VZ847`	Boolean Doob kernels: stationary marginals
`papers/stationary-weighting354`	`ARR-2026-4T35VQ6MZB8F99F4`	Boolean Doob kernels: moment transfer

Table 2: Complete cross-reference of all 33 companion Yang–Mills and mathematical-physics folders in `THE-ERIKSSON-PROGRAMME` to live AIRR archive identifiers (plus this technical note `ARR-2026-7DYZAEKGJP8FPTJ1`, 34 total).

6 Reproducibility, Citation, and Disclosure

Reproduce the full core build, oracle-only imports, axiom-output validation, and per-module Amos oracle at release `yang-mills-archive-v1.0` (commit `6b54eca695e96b3038ceda18276c4e28fb45983f`; on Windows pass `-c core.longpaths=true`):

```
git clone -c core.longpaths=true \
  https://github.com/lluiseriksson/THE-ERIKSSON-PROGRAMME.git
cd THE-ERIKSSON-PROGRAMME && git checkout yang-mills-archive-v1.0
lake exe cache get
lake build YangMillsCore AmosClosure \
  YangMills.RG.UniformPhysicalCoarseRankWallOracle
# Build all 46 imports of oracle_check.lean (matching .github/workflows/lean-build.yml):
grep -E '^import ' oracle_check.lean | awk '{print $2}' | xargs lake build
lake env lean oracle_check.lean > oracle-output.txt
python scripts/check_oracle_output.py --output oracle-output.txt
lake env lean AmosClosure/Oracle.lean
python scripts/check_consistency.py
```

Code is licensed `AGPL-3.0-or-later`; citation metadata live in `CITATION.cff` and `.zenodo.json`. Large language models assisted formalization and editing under author supervision; core claims are checked by the Lean 4 kernel with standard axioms only. Author Lluis Eriksson administers AIRR (https://airr.science/); no independent editorial admission or peer review is claimed.